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I. *On The Use of Anderson's Bridge for the Measurement of the Variations of the Capacity and Effective Resistance of a Condenser with Frequency.* *By* S. BUTTERWORTH, M.Sc., *National Physical Laboratory.*

RECEIVED JULY 1, 1921.

ABSTRACT.

An analysis of the effect of residuals and earth capacities in Anderson's Inductance-capacity bridge is made, and it is shown that if balances are obtained

(*a*) by balancing the bridge with *direct* currents ;

(*b*) by making the alternating current adjustments by means of a small series resistance (s') and parallel condenser (C') in the condenser arm ;

then the changes required in s' and C' to hold the balance at different frequencies are equal and opposite to the variations of the effective (series) resistance and capacity of the condenser with frequency.

The assumptions made in obtaining the above conclusions are that the residual inductances and resistances of the "non-inductive" arms of the bridge are invariable with frequency and that the resistance of the inductive arm varies as the square of the frequency. No knowledge of the absolute values of the residuals, &c., is required for the method.

The method is illustrated by results obtained with a condenser of capacity $0{\cdot}5\mu F$, and details are given showing how the chief experimental trouble, viz., drift in D.C. balance owing to temperature variations, may be overcome.

I.—Introduction.

ALTHOUGH Anderson's bridge has been used frequently in precision measurements of inductance and capacity and the effects of the residual inductances of the arms have been discussed,* a valuable property of this bridge seems to have been overlooked. An examination of the equations of balance of this bridge in which the effects of residuals are included shows that if the balancing adjustments are carried out in a particular manner, the only factors which necessitate a readjustment of balance at various frequencies of current are :

(*a*) the variation of the capacity of the condenser with frequency,

(*b*) the variation of the effective resistance of the condenser with frequency,

(*c*) the variation of the inductance with frequency.

The assumptions upon which the above conclusions are based are that the resistances and residual inductances (or capacities) of the "non-inductive" arms are invariable with frequency and that the resistance of the inductive arm has a term varying as the square of the frequency.

The observed variations of adjustment with frequency may therefore be used to determine the frequency variations of capacity and effective resistance of a condenser, the frequency coefficient of the inductance being supposed given. This frequency coefficient may be determined by a measurement of the self capacity of the inductive coil if the coil is of stranded wire. The absolute magnitudes of the residuals need not be known if the only quantities required are the *variations* of capacity and effective resistance of the condenser.

II.

Anderson's bridge is shown in Fig. 1*a* in which X_1, X_2, X_3, X_4 are resistances having small inductances, X_5 is an inductive coil and X_6 is the condenser under test.

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* Rosa and Grover, Bull. Bureau of Standards, Vol. I., p. 291 (1905).

If X_3 is zero the bridge becomes Maxwell's inductance-capacity bridge (Fig. 1(*b*)). The latter bridge may also be derived from Anderson's bridge by converting the "star" combination X_1, X_2, X_3 to its equivalent "mesh,"* $X'_1, X'_2, X'_3 = (X_3X_1+X_1X_2+X_2X_3)/X_2, X_1, X_3$ . . . . (1) the arm X'_3 acting as a shunt across the detector.

It will first be shown that small constant residuals in the arms X_1, X_2, X_3 convert to small constant residuals in the equivalent arms X'_1, X'_2, X'_3 and then the complete conditions of balance, including residuals, in Maxwell's bridge will be obtained.

Let $X_n = R_n + j\omega l_n \quad (n=1, 2, 3)$

Then

$$\begin{aligned} X'_1 &= X_1+X_3+X_1X_3/X_2 \\ &= R_1+R_3+j\omega(l_1+l_3)+(R_1+j\omega l_1)(R_3+j\omega l_3)/(R_2+j\omega l_2) \\ &= R_1+R_3+R_1R_3/R_2+j\omega\{l_1(1+R_3/R_2)+l_3(1+R_1/R_2)-l_2R_1R_3/R_2^2\} \end{aligned}$$

to the first order of small quantities

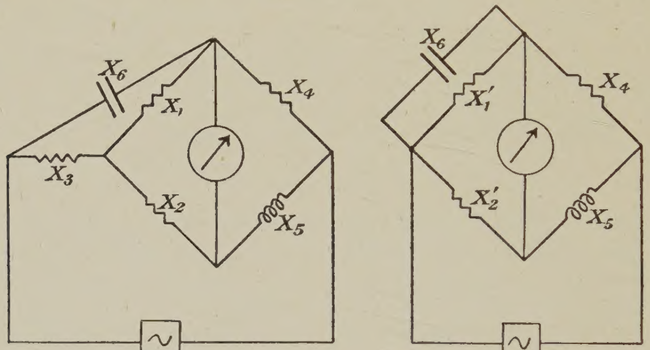

FIG. 1.—(*a*) ANDERSON'S BRIDGE. (*b*) MAXWELL'S BRIDGE.

Hence if l'_1, l'_2, l'_3 are the residual inductances of the equivalent arms X'_1, X'_2, X'_3

$$\left.\begin{aligned} l'_1 &= l_1(1+R_3/R_2)+l_3(1+R_1/R_2)-l_2R_1R_3/R_2^2 \\ l'_2 &= l_2(1+R_3/R_1)+l_3(1+R_2/R_1)-l_1R_2R_3/R_1^2 \\ l'_3 &= l_1(1+R_2/R_3)+l_2(1+R_1/R_3)-l_3R_1R_2/R_3^2 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

If l_1, l_2, l_3 are invariable with frequency, then to the first order of small quantities l'_1, l'_2, l'_3 are also invariable.

For analytical simplicity it is convenient in certain cases to replace residual inductances by residual capacities. Thus if l is the residual inductance of a resistance R, the equivalent residual capacity acting as a shunt is

$$c = -l/R^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

III.—*Conditions of Balance in Maxwell's Bridge.*

Maxwell's bridge is redrawn in Fig. 2 with the residuals included.

R_1 is a resistance having residual inductance l; R_2 is an inductive coil of inductance L; G_2 is a conductance with residual capacity c; G_1 is a conductance unting the condenser C, its residual capacity being included in C. The condenser has leakage conductance g.

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* G. A. Campbell, Trans. Am.I.E.E., Vol. XXX., p. 891 (1911).

If the bridge is earthed at the point E the earth capacities (assumed to be fficiently represented by condensers connecting the branch points to earth) are her included in c, C or are thrown across the whole bridge.

The vector equation of balance when using currents of frequency $\omega/2\pi$ is

$$(G_1+g+j\omega C)/(G_2+j\omega c)=(R_2+j\omega L)/(R_1+j\omega l) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

om which the two conditions of balance are

$$(G_1+g)R_1-\omega^2 lC=G_2R_2-\omega^2 Lc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$CR_1+l(G_1+g)=cR_2+LG_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

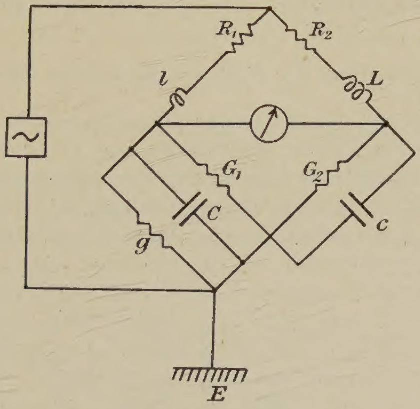

FIG. 2.—MAXWELL'S BRIDGE, INCLUDING RESIDUALS.

Now let the parallel combination of capacity C shunted by conductance g be placed by its equivalent series combination of capacity K in series with resistance so that

$$g=s\omega^2K^2/(1+\omega^2s^2K^2),\ C=K/(1+\omega^2s^2K^2). \quad . \quad . \quad . \quad . \quad . \quad (7)$$

ıce normally $s\omega K$ will seldom exceed 0·001, we may write to an accuracy 10^{-6}

$$g=s\omega^2C^2,\quad K=C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

d the two conditions of balance are now

$$G_1R_1+\omega^2(C^2sR_1-lC+Lc)=G_2R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$CR_1=LG_2-lG_1+cR_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

oducts of residuals being ignored.

IV.

Apart from the variations of C and s with frequency all the factors in the ove equations are invariable with frequency except R_2 and L, and for a stranded re coil we may write

$$L=L_0(1+\alpha\omega^2),\ R_2=R_0(1+\beta\omega^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Suppose now that the condition

$$G_1R_1=G_2R_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

has been satisfied by balancing with *direct* currents. Then, using (11) and (1 and ignoring products of small quantities, (9) and (10) become

$$C^2 sR_1 - lC + L_0 c = G_2 R_0 \beta \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1$$

$$CR_1(1 - a\omega^2) = L_0 G_2 - lG_1 + cR_0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1$$

If, therefore, the alternating current balance is secured by having a sma adjustable air condenser C' in parallel with C and a small adjustable resistance in series, equations (13) and (14) show that *for all frequencies*, $s + s'$ *and* $C(1 - a\omega^2) + C$ *are constant.*

The variations required in s' and C' to restore balance at the various frequencie will therefore be equal and opposite to the variations of s and $C(1 - a\omega^2)$.

V.—*Bridge Details for Effective Resistance Measurements.*

The bridge arrangements when used for the measurement of the variations i effective resistance of ½ microfarad condensers are shown in Fig. 3. The arm corresponding to X_1, X_2 in Fig. 1(*a*) were similar non-inductive coils each of 100 ohm The inductive arm was made up of an inductance coil R_2, L and a non-inductive 1 ohm resistance. The coil L consisted of a single layer 7/36 stranded wire coil

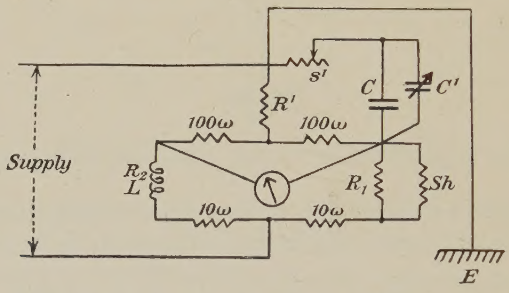

FIG. 3.—ANDERSON'S BRIDGE FOR EFFECTIVE RESISTANCE OF ½ μF CONDENSERS.

inductance 780 microhenries and resistance approximately 2·6 ohms. Its sel capacity, including the connecting leads, was 20 micro-microfarads, so that its increa in effective inductance at a frequency of 2,000 cycles per second was 3/10⁶. Th arm adjacent to the inductive arm was made up of a non-inductive 10-ohm coil ar a non-inductive copper coil R_1 of resistance slightly greater than that of the coil R The coil R_1 was shunted by a resistance box (*Sh.*). The inductive coil R_2 and th copper coil R_1 were enclosed in the same wool-lined box so that their temperatu variations would be as nearly as possible identical, the function of R_1 being to balan any changes of resistance due to temperature variations of R_2. (*See* Section VII.)

C was the condenser under test of nominal value ½ microfarad. It was shunte by an adjustable air condenser C' whose capacity ranged from 60 to 260 micr microfarads, and the combination was in series with a constant inductance rheost s' ranging from 0·1 to 1·1 ohm which could be read to 0·001 ohm. The resistan R' corresponding to X_3 in Fig. 1(*a*) was for purposes of preliminary balancing. could be adjusted by steps of 0·01 ohm to 100 ohms.

VI.—*Modifications for the Measurement of Variations of Capacity with Frequency.*

The above arrangements required modification when the capacity variations were being studied. Any inductance in the condenser arm will introduce a frequency variation in the effective capacity of this arm, so that the presence of the constant inductance rheostat s' in the condenser arm is a disadvantage. However, the direct current balancing condition is no longer of importance, so that if s' is transferred to the inductive arm R_2 and the fine adjustments are made by s' in R_2 and s' in parallel with C, then, since in (14) we may replace R_0 by R_2 to the order of accuracy required, small variations of R_2* will only introduce second order errors in the capacity equation as R_2 is associated only with the residual capacity c. The capacity arm C may then be made to include only the condenser and its leads, and only a small correction is required for the inductance of the leads.

These modifications were accordingly used when capacity variations were being measured.

VII.—*Temperature Variations.*

The chief source of trouble in the effective resistance measurements was the difficulty experienced in holding the direct-current balance throughout the alternating-current adjustments, the balance tending to drift because of the large temperature coefficient of the resistance of the inductive coil in R_2. This difficulty was largely overcome by the use of the balancing coil R_1 of Fig. 3, both R_1 and R_2 being shielded from stray temperature variations by enclosing them together. It was found necessary also to shield the leads to these coils. For similar reasons the current supplied to the bridge, whether direct or alternating, was made to have the same R.M.S. value. Even with these precautions a small drift in balance often occurred so that observations were taken in a definite order, and a correction applied for want of D.C. balance. An examination of equation (9) shows that a departure from condition (12) introduces an error in s proportional to $1/\omega^2$, and is thus more serious at the lower frequencies. For this reason it was found advisable to take the lower frequency observations immediately after securing the D.C. balance. Also by deliberately throwing the D.C. balance out of adjustment and observing the change in s' required to re-balance with alternating currents of known frequency, the correction for a given want of balance could be determined. The method is illustrated by the following example.

VIII.—*Example Showing Correction for Drift in D.C. Balance.*

$Sh \doteqdot 400$ ohms. $R_1 \doteqdot 3$ ohms.

An increase of 10 ohms in Sh causes an increase in deflection of the D.C. galvanometer of 35 divisions, and to restore the A.C. balance at a frequency of 500 $\sim$ per second requires a diminution in s' of 0·129 ohm.

Hence for a want of balance represented by 1 division of the scale of the D.C. galvanometer the correction on $s'=0{\cdot}0037$ ohm.

---

* It is essential, however, that R_2 shall remain of constant *inductance* throughout the adjustments, so that s' must be of the type specified.

The correction is applied as in the following table :—

Time. Min.	D.C. observations.		A.C. observations.		Deduced.	
	Source.	Galvo. deflections.	Frequency.	s' ohms.	Galvo. zero.	Want of balance.
0	+	+2·0	...	...	...	...
1	—	+4·0	...	...	+2·8	...
2	+	+1·0	...	...	...	—1·8
4	...	...	500	0·520*	...	...
6	+	—5·0	...	...	...	—7·5
7	—	+10·0	...	...	+2·5	...
8	+	—5·0	...	...	...	...

Want of balance at moment of A.C. observation$=$—4·6.
Therefore corrected $s'=0{\cdot}520-0{\cdot}0037\times 4{\cdot}6$
$=0{\cdot}503$ ohm

IX.—*Typical Results for the Change in Effective Resistance of a 0·5 μF Condenser.*

Preliminary adjustments are made by R' and Sh, and the remaining adjustmen at *all* frequencies by s' and C' only, observations being taken as in the above tabl The following results were obtained in a typical case :—

Condenser I (0·5 μF).
Bridge current$=$30 milliamperes.

Frequency ∾ per second...	500	800	1,000	1,600	2,00
s' ohms	0·503	0·597	0·625	0·675	0·68
s ohms	0·287	0·193	0·165	0·115	0·10
Power-factor ωsC	$10^{-4}\times 4{\cdot}5$	4·8	5·2	5·8	6·6

The values of s have been obtained from the known value of s for this condens at a frequency of 1,600 ∾ per second,[†] and the law $s+s'=$constant for all frequencie

X.—*Typical Results for the Variation in Capacity of a 0·5 μF Condenser.*

The rheostat s' is removed from the condenser arm, and placed in series in th arm R_2. After preliminary adjustments by R' and Sh the fine adjustments an adjustments for different frequencies are made by s' and C' only. The followin results hold for condenser I. :—

Frequency. ∾ per sec.	C'* $\mu\mu F$.	$C-C_{1000}$ Uncorrected $\mu\mu F$.	Correction $\mu\mu F$.	$C-C_{1000}$ corrected $\mu\mu F$.
500	132	+46	+3	+49
675	154	+24	+2	+26
830	164	+14	+1	+15
1,000	178	0	0	0
1,170	185	—7	—1	—8
1,510	193	—15	—3	—18
2,000	193	—15	—10	—25

The correction in the fourth column is for the variation of the inductive coil with frequency and for the residual inductance of the leads to the condenser. As regards the latter, if λ is the inductance of the leads, the effective capacity of the whole condenser arm is $C(1+\omega^2\lambda C)$. In the above observations $\lambda=0{\cdot}38$ microhenry, so that using $C=\frac{1}{2}\mu F$,

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* Mean of source and telephone reversals.

[†] Butterworth, Proc. Phys. Soc., Vol. XXXIII, p. 346, 1921.

$$\omega^2\lambda C=30\times10^{-6}$$

at a frequency of 2,000 cycles per second. Now the correction due to L (section 5) is 3×10^{-6} and (equation 14) acts oppositely to the λ correction, so that the whole correction is 27×10^{-6} at 2,000 ∼ per second. For a $\frac{1}{2}\mu F$ condenser this leads to a negative correction of 13·5 $\mu\mu F$. The corrections for the lower frequencies follow from the square law. The corrected variations are plotted against frequency in Fig. 4. It is seen that the smoothed curve never differs by more than five parts in a million from the observed values.

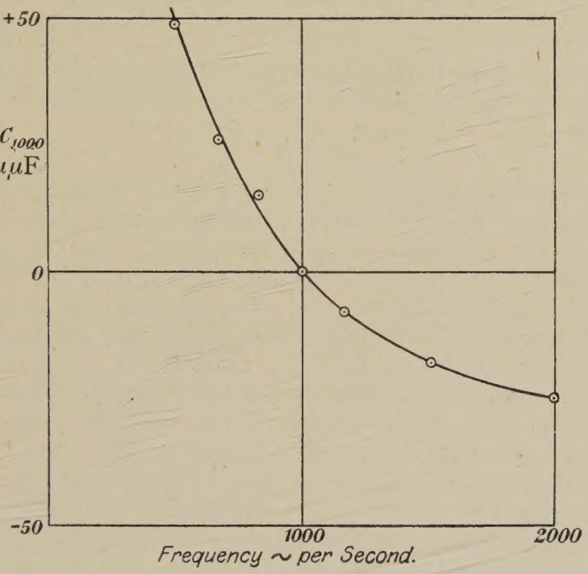

FIG. 4.—CHANGE OF CAPACITY OF $\frac{1}{2}$ μ F CONDENSER WITH FREQUENCY.

DISCUSSION.

Prof. FORTESCUE, referring to the typical results for the change in effective resistance of a condenser given in section IX. of the Paper, remarked that telephone engineers usually assume that the power factor of a condenser is constant, and would be specially interested to see that this assumption is incorrect. He enquired as to the type of condenser used.

Dr. D. OWEN congratulated the author on his valuable contribution, and expressed a belief that his methods would be adopted as the standard practice. It would be of interest to apply them to types of bridge other than that of Anderson. He asked if a sufficient number of condensers had been tested to determine whether the frequency coefficient of mica condensers is a definite quantity, or varies from one condenser to another like the temperature coefficient.

Mr. S. BUTTERWORTH, in reply to the discussion, stated that the results quoted in illustration of the method were obtained with a standard mica condenser. The power factors of four other mica condensers were also obtained, and the results varied between 0·0001 and 0·0007. Only one condenser was tested in regard to variation of capacity with frequency. However, a large number of mica condensers have been measured by H. L. Curtis (Bureau of Standards Circular, S.137), and his results are regarded as standard. The results of Curtis depend on the perfection of an air condenser of 0·1 μfd. capacity.

II. *Notes on Earth Capacity Effects in Alternating-Current Bridges.* By S. BUTTERWORTH, *M.Sc., National Physical Laboratory.*

RECEIVED MARCH 23, 1921.

ABSTRACT.

It is shown that an earth capacity acting at any point in the arm of a bridge may be replaced by two earth-impedances acting at the ends of the arm together with an impedance in series with the arm. By integration the result is extended to small distributed capacities.

Two methods are given for the elimination of the error due to the end impedances. Complete elimination can only be obtained by the use of shields connected to the ends of the bridge arm.

I.

IN precision measurements with alternating-current bridges it is always necessary to eliminate the effects due to the capacity of the various portions of the apparatus with regard to earth. The effect of these capacities is that the network with which we are dealing is not that composed of the visible arms, but includes other arms connecting various points to earth, and therefore different parts of the bridge to each other. Unless these earth paths are allowed for, different balances will be obtained as the bridge-earth conditions are changed. If it may be assumed that the earth capacities are sufficiently represented by condensers connecting the branch points of the bridge to earth, then there are two methods of eliminating the effect of these capacities in bridges which are reducible to one of the Wheatstone bridge type. These methods are described in Sections II. and III. below. The remainder of the Paper deals with the validity of the assumption in regard to earth capacities acting in the middle of an arm and in regard to distributed capacities. It is shown that the elimination is not complete by the two methods adopted. The error due to the incomplete elimination is not, however, serious if the parts of the apparatus are not brought too near to large masses of conductors connected to (or near to) earth, as the earth capacities tend rapidly to a constant value, and therefore behave as practically constant residuals in various bridge measurements. The effect is, therefore, that a different interpretation must be given to the residuals from that usually assumed. The residual inductance of a resistance coil, for example, is not merely that due to the magnetic field and its own intercapacity, but also includes a term which depends on the distributed capacity of the coil to earth. If in all measurements the coil is made to have the same position with regard to earth no discrepancy will appear when the coil is used in different bridges. This portion of the residual must, however, be taken into account in calculated standards of high resistance.

II.—*Elimination of Earth Capacities by the " Wagner " Earthing Device.*

The Wagner earthing arrangement as applied to a generalised Wheatstone bridge is shown in Fig. 1. a, b, c, d are the impedances of the arms of the bridge, while e and f are impedances so chosen that the relations $e/f=a/b=c/d$ may be approximately satisfied. The junction between e and f is connected to earth. Assuming that earth capacities or leakages may be represented by impedances connecting the branch points to earth, this earth point causes the paths of the earth currents to pass from the points 1, 2, 3, 4 to the point 5. The paths 5–1, 5–3 will merely modify the impedances e and f, but the paths 5–2, 5–4 will produce currents disturbing the lancing conditions if the points 2, 4, 5 are at different potentials. The procedure nsists in bringing the points 5, 2, 4 to the same potential by successively adjusting main bridge and the impedances *e* and *f* until the detector indicates balance ether connected to the point 2 or the point 5.*

III.—*Correction for Earth Capacities by Alteration of Earth Point.*

Let the symbols in Fig. 2 denote admittances, α, β being the admittances of the rth paths.

Let two balances be made (*a*) with the admittance *c* infinite, (*b*) with the ad-ttance *c* finite. Suppose the balances secured by adjustment of *A*. Then, if A_1 d A_2 are the two adjustments of *A*, while $A_0=BX/Y$, we have

$$\frac{A_1+\beta}{B+\alpha}=\frac{X}{Y}=\frac{A_2+\mu\beta}{B+\mu\alpha}$$

which

$$\mu=1+\frac{A_2+B}{c}$$

Hence, eliminating β/α

$$\frac{X}{Y}=\frac{A_2-\mu A_1}{B(1-\mu)} \text{ or } A_0=\frac{A_2-\mu A_1}{1-\mu}$$

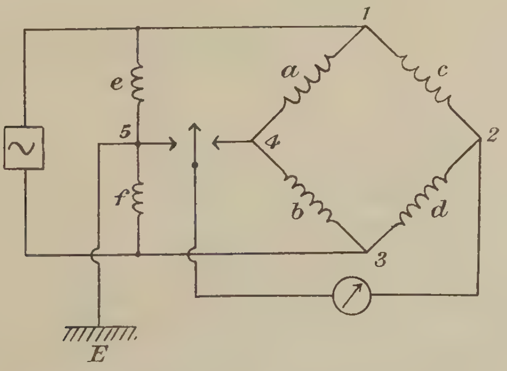

FIG. 1.—WAGNER EARTHING ARRANGEMENT.

If *c* is chosen to consist of an arm equivalent to A_2 and *B* in parallel, $\mu=2$ and

$$A_1=A_0+(A_2-A_1)$$

If a_0, a_1, a_2 are the corresponding impedances and a_1-a_2 is small,

$$a_1=a_0+a_2-a_1,$$

, regarding a_0 as the unknown,

$$a_0=2a_1-a_2.$$

The change in adjustment from a_1 to a_2 thus doubles the error, and the method, erefore, also gives a measure of the earth capacity effect. It must be remembered in applications that a_1 and a_2 are vector impedances, and therefore each adjustm is a double adjustment.

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* The device was originally applied for the purpose of eliminating the "head" effect when ing a telephone as detector in the comparison of small capacities by the Wien bridge. It has en extended to other bridges, particularly the Carey-Foster bridge, by Mr. D. W. Dye. The heme given by G. E. Moore (Electrician, June 17, 1921) is defective.

This method suffers from the following disadvantages as compared with th Wagner method :—

(*a*) The body must be kept in the same position in both adjustments when telephone is being used as detector.

(*b*) It is impossible in certain cases to secure a zero balance. This occurs som times in the measurement of small capacities by the Carey-Foster method.

The second difficulty may be overcome in the Carey-Foster bridge by using a auxiliary condenser for the zero balance and then placing the condenser to be measure in parallel with the auxiliary condenser.

It has the following advantages over the Wagner method :—

(*a*) Two adjustments only are necessary instead of a series of adjustments.

(*b*) An estimate may be made of the errors due to earth capacities.

The same auxiliary apparatus is required as in the Wagner method, the seri arms *e* and *f* of the Wagner arrangement becoming paralleled to form *C* in the arrang ment of Fig. 2.

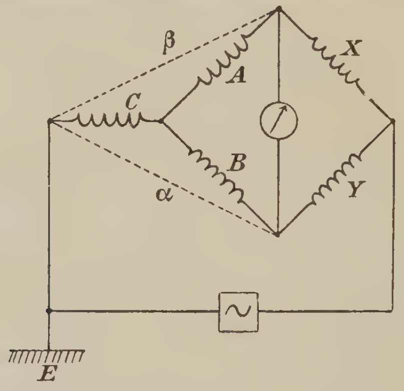

Fig. 2.—Correction for Earth Capacity by Alteration of the Earth Point.

IV.—*Earth Capacities in Compounded Arms.*

The assumption that earth capacities may be represented by earth paths fro the branch points of the bridge will now be examined.

If an arm is made up of several elements, appreciable earth capacities may b located at the junctions of these elements. This is particularly the case when the is a fairly large mass of metal at the junction, as in the case of condensers in seri with a resistance or inductance. Even terminals may in certain cases produce a appreciable effect.[*]

Let an arm consist of two series elements of impedance α, β having an eart path of impedance γ at their junction. Transformation (*A*)[†] converts the arrang ment to an arm of impedance $\alpha+\beta+\alpha\beta/\gamma$, with two end earth paths of impedanc

---

[*] A terminal approximating in size to a sphere of 1 cm. radius has an earth capacity of th order $1 \mu\mu F$.

[†] Transformations (*A*) and (*B*) are described in the author's Paper on " Capacity and Edd current Effects in Inductometers." (Proc. Phys. Soc., Vol. XXXIII., p. 312, 1921.)

$\beta+\beta\gamma+\gamma\alpha)/\beta$, $(\alpha\beta+\beta\gamma+\gamma\alpha)/\alpha$ respectively at the outer ends of α and β. If one the methods described has been used to eliminate earth capacities, the effects of e end impedances are eliminated, but there is an uneliminated portion behaving a residual $\alpha\beta/\gamma$ in series in the arm $\alpha+\beta$.

Thus, let α, β be two " non-inductive " resistances connected through a massive erminal.

If c is the earth capacity of the terminal, R_1, R_2 the values of the resistances, e uneliminated earth effect behaves as a residual inductance, cR_1R_2.

With resistances of 1,000 ohms, and with $c=1\mu\mu F$, this residual inductance $1\mu H$.

If α is a coil of resistance R and inductance L, while β is a condenser of capacity , then at a frequency $\omega/2\pi$

$$\alpha\beta/\gamma=\frac{c}{C}(R+j\omega L)$$

r the uneliminated portion of the earth capacity at the junction introduces an error the power factor of the condenser C of an amount ωcR, and causes the inductance o appear too large by an amount cL/C.

In illustration, let $c=50\mu\mu F$, $R=100$ ohms, $\omega=10^4$. The error in power factor then $5/10^5$. Large earth capacities at the junction should, therefore, be avoided y shielding the condenser and connecting the shield to that terminal of the branch oint which is connected to the condenser.

V.—*Distributed Earth Capacities.*

Suppose the earth capacity to be small and let it be distributed uniformly hroughout the arm. Suppose also that the impedance is distributed uniformly.

Then if Γ is the whole earth admittance and X the whole impedance of the arm f length l, the earth admittance $\Gamma\delta x/l$ of an element δx situated at a distance x long the arm contributes a series impedance $\Gamma X^2x(l-x)\delta x/l^3$, and end earth admit-ances $\Gamma(l-x)\delta x/l^2$, $\Gamma x\delta x/l^2$ to the arm, these results being obtained by transformation A) retaining only first order quantities. With the same approximation the earth aths of all the elements may be treated as acting independently so that by integra-ion the whole distributed earth admittance is represented by a series impedance of mount $\frac{1}{6}\Gamma X^2$, and two equal end admittances each of amount $\frac{1}{2}\Gamma$. The former is quivalent to a negative admittance $\frac{1}{6}\Gamma$, acting as a shunt across the arm.

We conclude that the uneliminated portion of the distributed earth capacity ehaves as a negative residual capacity in parallel with the arm of an amount equal o $\frac{1}{6}$ the whole earth capacity.

VI.—*Calculation of Earth Capacity.*

Let a resistance be made of flat strip forming a loop of length $2a$, width $2b$ Fig. 3). If b/a is small the system may be treated as an elliptic disc of semi-axes and b. Now the capacity of an elliptic disc is $a/K(\sqrt{1-b^2/a^2})$ where $K(k)$ is the omplete elliptic integral of the first kind to modulus k. Further if the elliptic disc divided into narrow strips of equal width parallel to the minor axis, each strip will ave the same charge when the potential of the whole disc is raised, so that in the limiting case the capacity of the strip is uniformly distributed throughout the leng a. Further when b/a is small

$$K(\sqrt{1-b^2/a^2}) \doteq \log_e 4a/b$$

so that the whole earth capacity of the strip is $a/\log_e 4a/b$ electrostatic units.

In illustration let $a=10$ cm., $b=0{\cdot}5$ cm. Then the earth capacity is $10/4{\cdot}4$ cm $=2{\cdot}3$ cm. $=2{\cdot}6\mu\mu F$.

The above result may be applied to give an approximate value of the eart capacity of a resistance wound in the manner shown in Fig. 4, a form often adopted t avoid large self-capacities in high resistance coils. The resistance consists of sever separate loops wound on a mica cord.

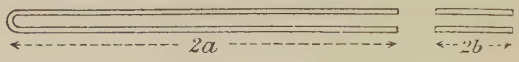

FIG. 3.

If each segment has a winding length of 3 cm. and winding width of 0·5 cm. th whole earth capacity of each segment would be about $1\mu\mu F$, so that if a 1,000 oh coil is made up of 10 segments each of 100 ohms, the whole earth capacity wou be about $10\mu\mu F$. When measured on a bridge in which end capacities are eliminate the uneliminated portion would contribute an inductance of the order $1{\cdot}7\mu H$.

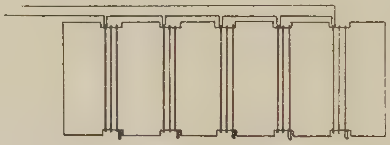

FIG. 4.

VII.—*Non-uniformly Distributed Capacity.*

The factor $\frac{1}{6}cR^2$ for the residual inductance introduced into a resistance having earth capacity c has been obtained upon the assumption of uniform distribution. If the distribution is denser towards the outer ends of the coil the abov estimate is too high, while if the capacity tends towards the middle, the estimate too low. If the coil is of the form of a circular disc the effect may be calculate Let a "non-inductive" coil be formed of two oppositely wound disc coils laid ove each other.

If a is the radius of the coils, the whole earth capacity is $2a/\pi$ electrostatic uni of which half is associated with one coil and half with the other. The density o distribution at distance r from the centre is proportional to $1/\sqrt{a^2-r^2}$. Now if is the length of wire in one coil, and x the length of wire between radii r and $l/x=a^2/a^2-r^2$, so that the density of distribution at a wire distance x from the out end is proportional to $x^{-\frac{1}{2}}$. Hence we write for the capacity per unit length Ax^{-} where since $A\int_0^l x^{-\frac{1}{2}}dx$ is the whole capacity of one coil (a/π), $A=a/2\pi\sqrt{l}$.

Again by the method of section 4, if ρ be resistance per unit length, the residual luctance due to the earth capacity of one coil is $\rho^2 A\int_0^l \sqrt{x}(l-x)dx$ $\frac{4}{15}\rho^2 A l^{\frac{5}{2}}=\frac{2}{15}\frac{a}{\pi}R^2$, R being the total resistance of one coil.

The equivalent outer end capacity is

$$\frac{A}{l}\int_0^l (l-x)dx/\sqrt{x}=\frac{4}{3}A\sqrt{l}=\frac{2}{3}\frac{a}{\pi}$$

d the equivalent inner end capacity is $\frac{1}{3}\frac{a}{\pi}$

Now let the two oppositely wound coils be connected at their inner ends. The rth capacities of the combination are equivalent to three earth capacities each of agnitude $\frac{2a}{3\pi}$ joining the ends and centre to earth together with a residual inctance $\frac{a}{15\pi R^2}$ (R now referring to the resistance of the whole combination).

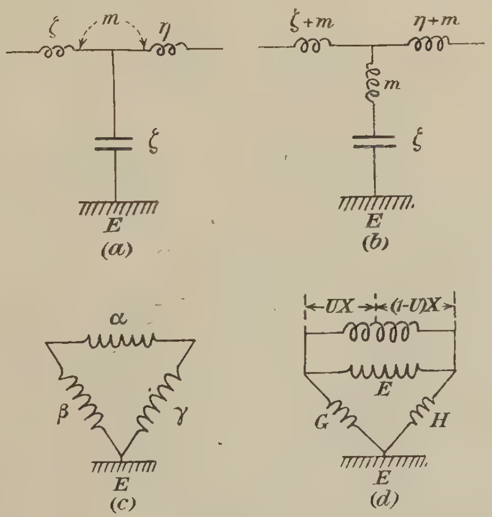

FIG. 5.

y transformation (A) the central earth capacity converts to a series inductance $\frac{a}{\pi R^2}$, and two end capacities each of magnitude $\frac{1}{3}a/\pi$. Hence writing c for the hole earth capacity so that $c=2a/\pi$, the whole effect is equivalent to a residual nductance $7cR^2/60$, together with two equal end capacities $\frac{1}{2}c$.

If the outers had been joined, the final system would have been a residual nductance $7cR^2/30$, and equal end capacities $\frac{1}{2}c$.

The effect of interchanging the end connections is thus to double the residual nductance effect.

VIII.—*Distributed Earth Capacities in Inductive Coils.*

Let an earth path of impedance, ζ, be present between two coils having impedances ξ, η and mutual impedance m {(*a*) Fig. 5}. By transformation (*A*) the mutual impedance, m, may be removed {(*b*) Fig. 5}, and then by transformation (*B*), the mesh α, β, γ {(*c*) Fig. 5} is obtained in which

$$\alpha=\xi+\eta+2m+\frac{(\xi+m)(\eta+m)}{\zeta+m}$$

$$\beta=\xi+\zeta+2m+\frac{(\xi+m)(\zeta+m)}{\eta+m}$$

$$\gamma=\eta+\zeta+2m+\frac{(\eta+m)(\zeta+m)}{\xi+m}$$

If ζ is large compared with ξ, η, m, then to the first order

$$\alpha=\xi+\eta+2m+\frac{(\xi+m)(\eta+m)}{\zeta}$$

which (again to the first order) is equivalent to the original two coils paralleled by an admittance

$$-\frac{1}{\zeta}\,\frac{(\xi+m)(\eta+m)}{(\xi+\eta+2m)^2}$$

while

$$\frac{1}{\beta}=\frac{1}{\zeta}\,\frac{\eta+m}{\xi+\eta+2m}\,,\quad\frac{1}{\gamma}=\frac{1}{\zeta}\,\frac{\xi+m}{\xi+\eta+2m}$$

In the case of distributed earth capacities there are an infinite number of paths such as ζ, and the above procedure may be applied to each, as to the first order of small quantities each path acts independently.

For an element of length $d\lambda$ of the coil, $1/\zeta$ will be of the form $Zd\lambda$ where Z is the earth admittance per unit length of the coil in the neighbourhood of the element. If X is the whole impedance of the coil, $X=\xi+\eta+2m$, so that, writing $\xi+m=UX$ the whole distributed earth admittances may be replaced by a shunt of admittance F, together with two end earth admittances G, H, F, G, H having the values

$$F=-\int ZU(1-U)d\lambda$$

$$G=\int X(1-U)d\lambda$$

$$H=\int ZUd\lambda$$

With pure earth capacities Z is of the form $j\omega C$ while U is a complex quantity, being the ratio of the potential difference up to the point λ to the whole potential difference between the ends of the coil. Hence except with special types of winding the equivalent end earth admittances and shunt admittance replacing a pure distributed capacity are not pure capacities, but rather behave as capacities having leakage.

IX.

A similar theory may be developed in regard to distributed intercapacities. theory is sufficiently illustrated by Fig. 6.

A single pure capacity C {(*a*) Fig. 6} is replaced by three imperfect capacities *b*). The capacities C_2, C_3 are then transformed to the capacity system shown in Fig. 6, so that to the first order this system of six leaky condensers is sufficient to resent the distributed capacity effects of two inductive coils.

For a single coil suppose the two coils to have a common point. The system n reduces to (*d*) Fig. 6. Finally the two condensers joining the ends to the common

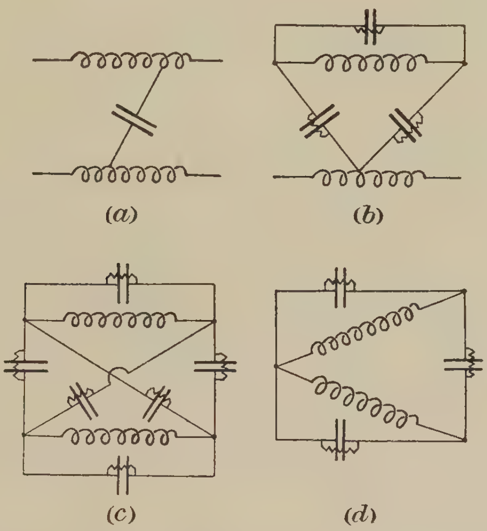
(*a*) (*b*)
(*c*) (*d*)

FIG. 6.—REPLACEMENT OF PURE INTERCAPACITY BY END SYSTEM OF LEAKY CONDENSERS.

nt are replaced by their equivalent end condensers, giving a single imperfect idenser across the whole coil, replacing the distributed pure capacity.

X.

The general conclusions arrived at are that the ordinary methods of eliminating th capacities do not completely eliminate the earth effect, either if the effect is tributed or if appreciable earth capacities are introduced in the middle of an arm. long as the whole arm remains a single unit in various bridge measurements no crepancies will arise unless the earth condition of the arm is considerably altered. however, the arm is split in transference from one bridge to another, discrepancies y appear with high resistances and small capacities. By providing shields in se cases, and arranging that the shields are connected to that terminal of the istance or condenser which is connected to a branch point in the measurements ought to be possible to make the observations agree within the limits of errors of ervation.

DISCUSSION.

Dr. A. RUSSELL remarked that the Paper contained much valuable matter and called for e study. He questioned whether it was justifiable at such high frequencies to lump the re-ıals or to leave mutual capacity out of consideration. In the examples given, *e.g.*, in section VI, of the Paper, nothing was said as to the distance from earth of the conductors whose ear capacity was under consideration. Electrical engineers are much engaged upon the proble of protecting telephone circuits from disturbance by power circuits, and find that the mutu capacity of such circuits is far more important than their mutual inductance.

Mr. Guild referred to the method of measuring earth capacity by doubling the error, d scribed in section **III.** of the Paper, and asked whether it was legitimate to assume that the ca city in question was unaffected by the operation of altering the connections.

Dr. D. Owen remarked that the Paper showed that standard resistances are subject to erro due to earth-capacity. Presumably standard inductances would be similarly affected.

Prof. Fortescue stated that in testing at frequencies of 20,000 ∿ or 100,000 ∿, errors 10 per cent. were common. Would the proposed methods apply to such tests and make it possib to reduce this error ?

Dr. Rankine referred to the disturbances due to external agencies, such as motors, whic are heard in thermionic valve amplifiers. Would the author's researches make it possible determine points in the apparatus, such that by connecting them to earth the disturbances referre to could be eliminated ?

Mr. S. Butterworth replied to the discussion as follows : The calculations of distribute earth capacities are given in order to indicate the order of magnitude of these quantities, and t effect of proximity to earth has been ignored for simplicity. In regard to the method of doubli the error, it has been tested by adding artificial earth capacities and consistent results have bee obtained. Errors due to earth capacities are only of importance in arms of high impedanc so that in calculated standards of inductance the earth capacity error only becomes appreciab when these are used at high frequencies. The errors in bridge measurements at radio frequenci should be considerably reduced by a proper choice of earth-point. In any case where it is desire to reduce the earth capacity effect in a particular arm the working principle is to bring t potential of that arm as near as possible to that of the earth.

I. *An Automatic Voltage Regulator.* *By* F. G. H. LEWIS.

RECEIVED JUNE 10, 1921.

ABSTRACT.

The Paper describes a method by which automatic voltage regulation to 0·15 per nt. may be obtained for such purposes as the operation of photometric standard lamps an ordinary outside supply varying by as much as 10 per cent. The lamp is placed ross an unbalanced Wheatstone bridge of which two opposite arms are composed of ngsten filament lamps. The increase of resistance of these lamps, due to the extra rrent passing through them when the outside voltage rises, causes a shift in the balance the bridge such that the voltage across the photometer lamp remains unaltered if the lues of the resistances in the arms be properly proportioned. The power taken is out 40 times that used in the regulated circuit.

IE object of the scheme described in what follows is to find a simple method by hich the fluctuations of voltage of an ordinary electric current supply may be ercome, and a steady voltage obtained for the purpose of running a glow-lamp otometer standard or any other purpose for which a steady voltage is required.

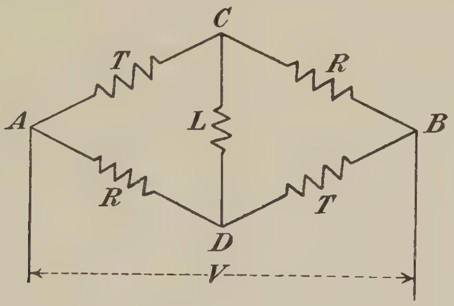

FIG. 1.—DIAGRAM OF CONNECTIONS.

The method depends on the fact that the resistance of a tungsten filament mp rises with increase of temperature of the filament in such a way that the rcentage increase in resistance is proportional to the percentage increase of current. is has been found experimentally by determining the change of current through a mp due to change of voltage. The following experimental results show the values current at various voltages for tungsten filament lamps of two different ratings :—

TABLE I.

100-volt Lamp.		200-volt Lamp.	
Voltage	Current	Voltage	Current
60	0·632	120	0·169
70	0·693	140	0·185
80	0·750	160	0·201
90	0·804	180	0·215
100	0·856	200	0·230
110	0·905	220	0·243

$dI/dV = 0{\cdot}60(I/V)$ so that $dR/R = 0{\cdot}67(dI/I)$

$dI/dV = 0{\cdot}61(I/V)$ so that $dR/R = 0{\cdot}64(dI/I)$

Now suppose an unbalanced Wheatstone Bridge be set up as in Fig. 1, where

T,T are resistances composed of tungsten filament lamps in parallel, while R are constant resistances of any ordinary type. L is a standard lamp, the curre through which it is desired to maintain constant, and V is a fluctuating supp voltage. It will be seen most clearly by means of a numerical example, how constant voltage may be maintained at L when V varies.

As an example, suppose that the outside voltage, which may vary from 220 260 volts, be applied at AB, and that it is desired to have 40 volts constant acro CD when the current flowing from C to D is 0·3 amperes. Then, when the volta across AB is 220 volts, the voltage from A to C is 90 volts and that from C to is 130 volts if the resistances in these two arms are respectively equal to the resi tances in BD and DA. If, then, i_1 is the current in CB, the current in AC is $(i_1+0\cdot$

Hence $90=T_1(i_1+0\cdot3)$ where T_1 is the resistance of T when the current throug it is $(i_1+0\cdot3)$.

Also $130=R\,i_1$.

Similarly when the outside voltage is 260, the voltage between A and C mu be 110 and that between C and B must be 150, in order to maintain 40 volts acro CD.

Hence $110=T_2(i_2+0\cdot3)$ where T_2 is the resistance of T when the current throug it is $(i_2+0\cdot3)$, for if the resistances of the arms AC and DB be equal at the low value of current, they will also be equal at the higher current, since both are com posed of lamps having the same current-resistance characteristic.

Also $$150=Ri_2$$

Hence $$T_2/T_1=11(i_1+0\cdot3)/9(i_2+0\cdot3)$$

and $$i_1/i_2=130/150$$

But since for a tungsten lamp $dR/R=a\ (dI/I)$ where a is about 2/3, it follo that

$$(T_2-T_1)/T_1=\frac{2}{3}\{(i_2+0\cdot3)-(i_1+0\cdot3)\}/(i_1+0\cdot3)$$

whence $$T_2/T_1=\frac{2}{3}(i_2+0\cdot3)/(i_1+0\cdot3)+\frac{1}{3}$$

Thus from the two expressions for T_2/T_1

$$\frac{2}{3}x^2+\frac{1}{3}x-\frac{11}{9}=0$$

where $$x\equiv(i_2+0\cdot3)/(i_1+0\cdot3)$$

This equation gives $$x=1\cdot127$$

and from this result, since $i_1/i_2=13/15$, we get $i_1=1\cdot43$, and hence $T=52$ a $R=91$ ohms.

Thus a regulated voltage of 40 volts with a current of 0·3 amps. may be obtaine with a total current of approximately 3 amps. on a variable circuit of nominal 240 volts.

This result may be somewhat improved by using, instead of constant resistanc in the arms CB and AD of the bridge, carbon filament lamps which have a sma negative temperature coefficient of resistance.

The device has been tried in this form with the following values of resistances :—

Arm *AC*—2 120-volt, 60-watt tungsten filament lamps, and 2 105-volt, 25-watt t.f. lamps in parallel.

DB—3 120-volt, 60-watt and 1 110-volt, 30-watt t.f. lamps in parallel.

AD and *CB*, each 5 200-volt, 16-c.p. carbon-filament lamps in parallel.

The voltages measured across the lamp terminals in *CD* with various voltages oss *AB* are given in Table II.

TABLE II.

Voltage across *AB*.	Voltage across *CD*.
260	68·0
250	68·2
240	68·2
230	68·1
220	67·9

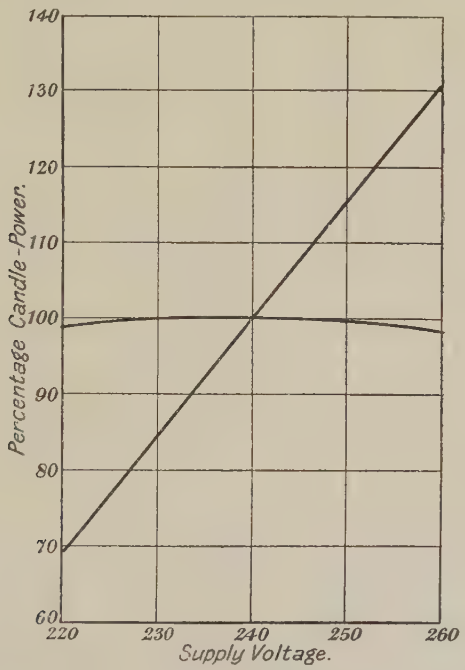

FIG. 2.—GRAPH OF RESULTS OBTAINED WITH REGULATOR.

Total current through circuit, 2·26 amps. ; current in *CD*, 0·20 amps.

It is suggested that this device should prove useful for a small photometric boratory where the expense of upkeep of a storage battery is excessive for the nount of photometric work to be done. As will be seen from the table above, a oltage regulation to within 0·2 per cent. can be obtained on a supply fluctuating y as much as 5 per cent. from the mean, with a circuit in which the total power sed is 40 times that needed for the photometer lamp.

The curves in Fig. 2 show the variation in the candle-power of a lamp operati (*a*) on the outside voltage as it varies from 220 to 260 volts, and (*b*) on the regulat voltage (over the same range of variation of supply voltage). In these curves value 3·7 has been used for the percentage variation of candle-power due to 1 per ce change of voltage.

The circuit is very easy and cheap to set up, and it can be readily adjusted trial and error to give the required result. It is desirable to have a switch or ser resistance in the regulated circuit, as the low resistance of the tungsten lamps wh cold causes a rush of current in this arm when the main current is first switched The device might also find application in meter testing, or in other cases where small current at a steady voltage is required.

DISCUSSION.

Dr. **Rayner** suggested that the apparatus described might afford a new type of standard E.M

Mr. F. E. **Smith** dissented from the view of the previous speaker, and showed that the volt which is to be kept constant is in fact a function of the resistance of the non-incandescent ar of the bridge. It can thus never be possible in such an arrangement to secure an absolut constant voltage, such as would be required for the purposes of a standard.

Prof. **Fortescue** said that the method produced a distinct gain in steadiness of voltage a would undoubtedly be appreciated in testing work.

Mr. J. W. T. **Walsh**, who read the Paper on behalf of Mr. Lewis, referred, in reply, to Fi of the Paper, which shows candle-power plotted against supply-voltage. The curve is v flat-topped, thus permitting of steadiness within very close limits, even for variations of sup voltage which would be considered as very large.

Communicated Remarks.

From Mr. T. **Smith**: Mr. Lewis' problem can be treated theoretically by considering o algebraic forms. Taking the resistances of the outer arms of the bridge in cyclic order as P, Q, R and the resistance on the regulated circuit to be G, the currents in these arms are evidently p portional to $S(Q+R)+G(R+S)$, $R(P+S)+G(R+S)$, $Q(P+S)+G(P+Q)$, $P(Q+R)+G(P+$ and $PR-QS$ respectively. If then a constant current C is to be maintained in the regula circuit the resistance in the first arm must be P when the current in this arm is

$$\frac{S(Q+R)+G(S+R)}{PR-QS}C$$

and similarly in the other cases. If P is the only variable resistance the regulation can only exact if the law followed by the filament is of the form

$$\text{Voltage}=\alpha\times\text{current}+\beta\times\text{resistance},$$

where α and β are constants. When these constants are known the proper resistances for bridge can be set up at once. The extension to the case when the resistance of more than one a depends upon the current is obvious from the above expressions. The external voltage is

$$\frac{PQRS\left(\frac{1}{P}+\frac{1}{Q}+\frac{1}{R}+\frac{1}{S}\right)+G(P+Q)\,(R+S)}{PR-QS}C$$

and the battery current is

$$\frac{(P+S)(Q+R)+G(P+Q+R+S)}{PR-QS}C$$

From these expressions it is evident that the power efficiency of the regulation will be low unl the control can be effected by the use of a material having a large value for the coefficient β. such a material were available it would evidently be desirable to employ it in the lamp of t regulated circuit, and with a suitable coefficient the need for the bridge would disappear. T condition would be exactly met if the resistance were proportional to the square of the voltage,

$$\frac{dR}{R}=2\frac{dV}{V}=-2\frac{dC}{C}$$

h may be compared with the author's experimental results. It is difficult to imagine a
rial with the property that the current decreases continuously over a certain range in con-
ence of a continuous increase in the applied voltage, but the author's scheme may enable
onstruction of standard lamps with a self-contained automatic voltage steadier.

From Mr. A. CAMPBELL: The thermal system of voltage regulation used by Mr. Lewis is not
l, except in so far as tungsten is employed as the heated material. The general system
described by the writer many years ago (Proc. Inst. El. Engs., Vol. XXX., p. 889, 1901),
the mathematical conditions were investigated for one case. The hot wires actually used
of nickel, platinum or copper, and very exact automatic regulation was obtained. A
olt lamp was exhibited regulated on a variable circuit of about 150 volts. In the discussion
e Paper referred to, Professor Callendar described his use of the system for potentiometer
oses; he was probably the first to employ the method. Mr. Lewis has made the method
easily available by putting ordinary tungsten filament lamps in place of specially
tructed resistances.

From the AUTHOR: I feel that Mr. T. Smith's mathematical treatment of the problem
much to the value of the Paper, but I would emphasise the fact that in any particular case
final adjustments are always more conveniently made by trial and error, since the
tances of the opposite arms of the bridge need not necessarily be exactly equal. As
Smith says, the efficiency of the device depends upon the quantity he calls β, and if
so large compared with α that the latter may be neglected, the need for the device would
pear. Unfortunately, as he remarks, the substance in which the current decreases as the
ied voltage increases is one somewhat difficult to imagine.

I regret very much that I was not aware of Mr. Campbell's previous description of the device
e the Institution of Electrical Engineers, or of Professor Callendar's use of it. It clearly
ars that the principle of the method was first employed by Professor Callendar, but I would
t out that the use of tungsten lamps instead of nickel resistances not only simplifies the
ng up of the device, as Mr. Campbell remarks, but it also has another more important result,
that of making the regulation independent of external temperature. Mr. Campbell
eved this by the use of compensating resistances placed in the same enclosures as the nickel
of his bridge, but any such arrangement is unnecessary in the case of tungsten lamps where
conductor operates at a temperature of something like 2000 deg. in a vacuum. It can be
n both theoretically and practically that the effect of even a 100-deg. change in the
erature of the surroundings is quite inappreciable in its effect on the temperature, *i.e.*, the
tance of the filament.

It thus seems that Dr. Rayner's suggestion is one well worth investigating further, and a
l form of the bridge, to be used on a circuit regulated to the nearest volt (as it might easily
could be made very handy and suitable for use with a potentiometer or for similar
dardising purposes, and should give a voltage which remains constant and unaltered by
erature to at least 0·01 per cent., and it is possible that a very much greater constancy
this might prove to be obtainable.

IV. *The Flow of Viscous Liquids through slightly Conical Tubes.* By Pr

A. S. Hemmy, *Government College, Lahore.*

Received May 5, 1921.

ABSTRACT.

A formula is obtained for the flow of a viscous liquid through a slightly conical tr by neglecting terms containing the square of the obliquity.

The approximation is shown to be justified by the agreement in the values of viscosity found experimentally with tubes of differing degrees of conicality.

The corrections to be applied to Poiseuille's formula for the flow of a visc liquid through a cylindrical tube have been considered in a number of Papers; this communication the case where the tube differs slightly from the true cylin is discussed.

I. Mathematical.

The radius a_2 of the tube at a distance z from one end is taken to $a(1+bz)$, and all terms containing the square of b are assumed to be negligible.

The stream lines are generators of circular cones having the same apex, that the velocity v at any point on a stream line at distance r from the axis can represented by

$$v=\frac{v_0 r_0^{\,2}}{r^2}=v_0(1-2bz) \quad . \; . \; . \; . \; . \; . \; . \; . \; .$$

where v_0 and r_0 are the values of v and r when $z=0$.

Likewise, if p is the pressure and G the value of dp/dz when $z=0$

$$\frac{dp}{dz}=G(1-2bz) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; .$$

Integrating along the whole length l of the tube

$$P=\int_0^l G(1-2bz)dz=G(l-bl^2)$$

or

$$G=\frac{P}{l}(1+bl) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; .$$

P being the pressure difference between the ends of the tube. The tangential str per unit length of the sides of the cylinder of radius r equals

$$2\pi r\eta\frac{dv}{dr}$$

where η is the coefficient of viscosity. The difference of tangential forces on the t surfaces of the cylindrical shell of thickness dr is therefore

$$2\pi\eta\frac{d}{dr}\left(r\frac{dv}{dr}\right)dr$$

This differs from the axial component by a quantity depending on b^2 and may erefore be taken as axial. We have, therefore,

$$2\pi\eta\,\frac{d}{dr}\left(r\,\frac{dv}{dr}\right)dr=-2\pi r dr\,\frac{dp}{dz}-2\pi r dr\sigma\,\frac{d\sigma}{dt}$$

here σ is the density and t the time, or

$$\eta\,\frac{d}{dr}\left(r\,\frac{dv}{dr}\right)=-rG(1-2bz)-\sigma r\,\frac{dv}{dz}\cdot\frac{dz}{dt}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From (1) $\frac{dv}{dz}=-2bv_0$ and (4) becomes

$$\eta\,\frac{d}{dr}\left(r\,\frac{dv}{dr}\right)=-rG(1-2bz)+2\sigma rbv_0\sigma$$

Substitute for v from (1), and, neglecting b^2,

$$\eta\,\frac{d}{dr}\left(r\,\frac{dv}{dr}\right)=-rG(1-2bz)+2b\sigma rv_0^2\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the third term, as b is small, we may substitute the value of v_0 as obtained y Poiseuille on the assumption that the tube is a true cylinder, viz. :

$$v_0=\frac{A}{4\eta_0}\,(a^2-r^2)$$

vhere $A=P/l$ and η_0 is the first approximation to the value of the viscosity on this ssumption, *i.e.*, introducing the usual correction for the kinetic energy of the entering iquid,

$$\eta_0=\frac{\sigma}{8l}\left(\frac{gH\pi m^4t}{v}-\frac{V}{\pi t}\right)\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

vhere m is the mean radius, H the head, and V the volume flowing through the ube in time t.

(5) now becomes

$$\eta\,\frac{d}{dr}\left(r\,\frac{dv}{dr}\right)=-rA(1+bl)(1-2bz)+2b\sigma r\,\frac{A^2}{16\eta_0^2}\,(a^2-r^2)^2$$

or

$$\eta\,\frac{d}{dr}\left(r\,\frac{dv}{dr}\right)=-rA(1+bl-2bz)+r\,\frac{b\sigma A^2}{8\eta_0^2}\,(a^2-r^2)^2\quad . \quad . \quad . \quad . \quad (7)$$

Integrating, we get

$$\eta\,r\,\frac{dv}{dr}=-\frac{r^2}{2}\,A(1+bl-2bz)+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r^2}{2}-\frac{2a^2r^4}{4}+\frac{r^6}{6}\right)+B$$

$B=0$, as stress$=0$ when $r=0$.

$$\therefore \quad \eta\,\frac{dv}{dr}=-\frac{r}{2}\,A(1+bl-2bz)+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r}{2}-\frac{a^2r^3}{2}+\frac{r^5}{6}\right)\quad . \quad . \quad . \quad . \quad (8)$$

Integrating again, we get

$$\eta v=-\frac{r^2}{4}\,A(1+bl-2bz)+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r^2}{4}-\frac{a^2r^4}{8}+\frac{r^6}{36}\right)+C$$

To find C, put $z=0$; then when $r=a$, $v=0$.

$$\therefore \qquad C=\frac{a^2A}{4}(1+bl)-\frac{b\sigma A^2}{8\eta_0^2}\,\frac{11}{72}\,a^6$$

$$\therefore \qquad \eta v_0=\frac{a^2-r^2}{4}A(1+bl)-\frac{b\sigma A^2}{8\eta_0^2}\,\frac{11}{72}a^6+\frac{b\sigma A^2}{8\eta_0^2}\left(\frac{a^4r^2}{4}-\frac{a^2r^4}{8}+\frac{r^6}{36}\right)\ldots(9)$$

The volume V of liquid which passes across the mouth of the tube in time is given by

$$V=2\pi t\int_0^a rv_0dr$$

or

$$\frac{\eta V}{2\pi t}=\frac{a^4}{16}A(1+bl)-\frac{b\sigma A^2a^8}{8\eta_0^2}\cdot\frac{1}{32}$$

whence

$$\eta=\frac{\pi ta^4P}{8Vl}\left(1+bl-\frac{b\sigma Pa^4}{16\eta_0^2l}\right)\quad\ldots\ldots\ldots\ldots\quad(10)$$

Put $P=g\sigma H$, where H is the head, and, including the correction for the kinetic energy of the liquid on entering the tube, we have, finally,

$$\eta=\left(\frac{\pi ta^4g\sigma H}{8lV}-\frac{\sigma V}{8\pi lt}\right)\left(1+bl-\frac{b\sigma^2gHa^4}{16\eta_0^2l}\right)\quad\ldots\ldots\quad(11)$$

II. EXPERIMENTAL.

By Prof. A. S. Hemmy and L. Sant Ram, M.Sc.

The need for the formula arose from the difficulty of obtaining perfectly cylindrical tubes for the usual laboratory method of determining coefficients of viscosity. So-called thermometer tubing is rarely perfectly cylindrical, and it is easier to obtain evenly conical lengths. It was desirable to determine how much a variation of radius affected the result. It was found that variations such as commonly occur exercise a marked influence. The correction introduced by formula (11) for a piece of tubing selected as being more cylindrical than the average was found to be nearly 2 per cent., *vide* exp. 9, Table II.

TABLE I.—*Dimensions of Tubes Used.*

Tube.	A	B	C	D	F
Length (l)	10·20	10·40	9·80	6·90	11·405
Mean radius (m)	0·10750	0·10720	0·07661	0·06500	0·10620
Radius at narrow end (a)	0·10632	0·10495	0·07560	0·06359	0·10593
Slope (b)	0·002154	0·004042	0·002767	0·006195	0·0003409

The usual method of correcting for variations in the radius of a tube is to replace l/m^4 in equation (6) by the integral of $dz/a_z{}^4$, but for an evenly conical tube this makes no change in the value of η to the first order of bl.

The formula was verified experimentally by one of us using tubes drawn to various degrees of conicality. A tubulated vessel was provided with a simple overflow arrangement to maintain the head constant at any desired value. Care was taken to keep the tubes horizontal. The head was adjusted in most cases to be just sufficient to maintain a steady jet. Tap water was used throughout. The tubes e calibrated and b determined by means of a mercury thread and a micrometer roscope. The value of m was determined by finding the weight of mercury ng the tube, and a calculated from the values of m and b.

TABLE II.—*Values of the Viscosity.*

ol. 1	2	3	4	5	6	7	8	9	10
o. of Exp.	Tube used.	H in cm.	V in c.c.	t in secs.	Temp. Cent.	b	η_0	η	η_{15}
1	A	6·040	250·0	154·8	15·0	0·002154	0·01252	0·01124	0·01124
2	B	6·040	250·3	164·2	15·0	0·004042	0·01351	0·01129	0·01129
3	B	6·830	250·3	150·4	15·0	0·004042	0·01366	0·01128	0·01128
4	D	13·215	70·24	93·0	15·0	0·006195	0·01304	0·01127	0·01127
5	C	10·248	89·00	101·0	15·4	0·002767	0·01213	0·01118	0·01129
6	B	7·493	88·11	49·7	14·8	0·004042	0·01385	0·01131	0·01125
7	C	11·050	62·50	67·5	14·9	0·002767	0·01236	0·01136	0·01133
8	C	11·131	104·55	109·8	16·0	0·002767	0·01193	0·01093	0·01121
9	F	5·852	93·57	61·2	16·5	0·000341	0·01108	0·01087	0·01128

In column 10, Table II., for the sake of comparison all results are reduced to the perature 15°C. by interpolation with the aid of Kaye and Laby's tables. Whilst values of η_0 in column 8 show wide variations, those in columns 9 and 10 justify approximations used in obtaining formula (11) and show that a good value of the cosity can be obtained with a conical tube.

DISCUSSION.

(Communicated after the Meeting.)

W. N. BOND, M.Sc. This communication consists of a criticism of certain parts of the ry of the Paper; and secondly, of an analysis of the author's experimental results.

The variation in the pressure gradient along the tube, given by equation (2) of the above er, was put forward without explanation, and does not appear to follow from the equations either purely turbulent or purely viscous flow through tubes.

For purely viscous flow we have $dp/dz \propto \eta V/r^4t$, and for purely turbulent flow $dp/dz \propto \sigma V^2/r^5t$, re the z axis is along the length of the tube, and r, V/t, σ and η denote the radius, volume sing per second, and density and viscosity of the liquid respectively. Then, adopting the tion of the Paper, $r=a(1+bz)$, where a is the radius of the tube at the narrow end. Hence, denotes the pressure gradient at the narrow end, we have $dp/dz=G(1-4bz)$ and $dp/dz=G$ $-5bz)$ in the cases of purely viscous and purely turbulent flow respectively. In the case con-red we may probably assume the first of these equations to be true.

According to equation (2) of the Paper $dp/dz=G(1-2bz)$.

In obtaining equation (11) of the Paper, pressure differences occurring in equation (10) replaced by the corresponding heads, or effective differences in level of the liquid. The ctive difference of head between the ends of the tube is deduced from the total head of liquid ployed, after subtracting a term due to the gain in kinetic energy of the liquid on entering tube.

This kinetic energy correction (which amounts in the case of some of the above experiments 0 per cent. of the total head employed) is, however, apparently not applied in obtaining the term in equation (11). Hence this last term (which itself in some cases affects the estimated ue of η by 10 per cent.) is considerably in error.

It is true that the equation (11) is in very good agreement with the experimental results; the agreement is not so close when the above changes are made. In any case, it will be iced that the values obtained for the viscosity are uniformly some 1 per cent. lower than those ally accepted.

If the usual correction due to the kinetic energy at entrance be assumed, we may suppose effective pressure difference between the ends of the tube to be partly opposed by viscous resistance, and partly by the resistance due to the negative acceleration of the liquid caus by the tube being divergent. Hence we may write

$$H-\frac{V^2}{\pi^2a^4t^2g}=\frac{8l\eta V}{\pi m^4g\sigma t}+kbl\frac{V^2}{\pi^2a^4t^2g}, \quad , \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

where $m=a(1+bl/2)$ = mean radius of tube ;
l = length of tube ;
H = total head of liquid employed.

It would be expected that k should have the value 4. And if that were so, the above equati would correspond to Prof. Hemmy's theory, except for the two changes mentioned above.

If we assume the usual values for the viscosity of water, and suppose that $g=978{\cdot}5$ (Lahor we can determine k from the experimental results, employing equation (I).

The following table gives the values of k thus obtained, together with the values of η calc lated from the experiments, using equation (I) and assuming firstly $k=4$, and secondly $k=6$.

TABLE I. (*Analysis of Prof. Hemmy's Results.*)

l	m	a	b	H	V/t	°C.	η calculated $k=4$	η calculated $k=6$	η assumed	k calc
10·20	0·10750	0·10632	0·002154	6·040	1·614	15·0	0·01163	0·01134	0·01142	5
10·40	0·10720	0·10495	0·004042	6·040	1·523	15·0	0·01193	0·01140	0·01142	6
10·40	0·10720	0·10495	0·004042	6·830	1·663	15·0	0·01191	0·01136	0·01142	5
10·40	0·10720	0·10495	0·004042	7·493	1·772	14·8	0·01199	0·01140	0·01149	5
9·80	0·07661	0·07560	0·002767	10·248	0·881	15·4	0·01147	0·01129	0·01130	[illegible]
9·80	0·07661	0·07560	0·002767	11·050	0·926	14·9	0·01166	0·01146	0·01146	[illegible]
9·80	0·07661	0·07560	0·002767	11·131	0·951	16·0	0·01124	0·01102	0·01110	[illegible]
6·90	0·06500	0·06359	0·006195	13·215	0·756	15·0	0·01180	0·01142	0·01142	[illegible]
11·405	0·10620	0·10593	0·0003409	5·852	1·528	16·5	0·01096	0·01089	0·01096	[illegible]

The fair agreement between the different values of k obtained might be taken to indic that its true value was of the order 5·7 : but the writer has been unable to find any theoreti reason why k should have such a value. It is just possible that the flow was on the verge of t bulence in the case of these experiments, and that this might cause a higher value of k than 4.

V. *The Estimation of the Radium Content of Radio-active Luminous Compounds.*

By E. A. Owen, *M.A., D.Sc., and* Winifred E. Fage, *B.Sc.* (*The National Physical Laboratory, Teddington.*)

Received August 15, 1921.

ABSTRACT.

(1) The absorption of γ-radiation in zinc sulphide has been measured by different methods, and in each case its value was found to be approximately the same.

(2) No indication was observed of the occurrence of an abnormal absorption when the radioactive salt was mixed with the sulphide.

(3) A table has been compiled giving the values of the ratio of the true to the apparent radium content for tubes of different diameters filled with radioactive luminous compound. This table applies to the case of a compound whose apparent density is 2·03 grammes per cubic centimetre.

(4) The values of the absorption of γ-radiation from radium have been measured in different salts, particular attention being paid to the barium salts with which radium is often found in combination.

The apparent radium content of a sample of luminous compound may be directly measured by means of any one of the γ-ray methods. To arrive at the true content, it is necessary to know how much of the radiation is absorbed in the salt. This absorption has been measured by Paterson, Walsh and Higgins,* who found that different values were obtained according to whether it was measured before or after mixing the radium with the responsive base—the base employed being hexagonal zinc sulphide. The value of the absorption coefficient of γ-radiation in the sulphide under the conditions of experiment was found to be about three times as much after mixing as it was before mixing with the radium. It was suspected that this difference was due to a selective effect which occurred in the one case but not in the other. This, however, was not definitely proved. The object of the present investigation was to obtain further information on this point. The absorption coefficient of γ-radiation in zinc sulphide has been re-determined by a number of different methods, and, in addition, data regarding the absorption of other salts have been obtained.

Apparatus.

The most convenient method of measuring the gamma ray activity of a radium salt is the Direct Method. This is one of the methods in constant use at the Laboratory, and has always been employed to measure the radium content of luminous compounds. The apparatus consists of a combined ionisation chamber and gold-leaf electroscope as shown in Fig. 1. The sample whose radium content is desired is placed some distance in front of the ionisation chamber. The distance between the sample and the chamber is decided by the amount of radium which the sample contains—about 30 divisions per minute is a convenient rate of fall of the leaf. The face of the ionisation chamber is covered with lead 0·5 cm. thick, so that only γ-radiation enters the chamber. To diminish the effect of stray radiation the windows of the electroscope are shielded with lead, and the walls of the chamber and the electroscope are made of lead 0·6 cm. thick. The amount of radium in a sample is measured by comparing it with a standard tube of known content, the experimental conditions during the observations being in each case exactly similar. The time taken by the leaf to fall over a constant number of divisions of the scale is measured both for the standard and for the sample. After correcting these times for the natural leak of the leaf system, which should be determined at the beginning and at the end of the observations, the corrected times are inversely proportional to the radium contents of the standard and sample, and thus the radium content of the sample is directly determined.

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* Proc. Phys. Soc., Vol. XXIX., p. 215 (1917).

METHODS OF MEASURING THE ABSORPTION OF THE RAYS IN THE SULPHIDE.

The absorption of the radiation in zinc sulphide has been measured in the following ways: (1) The sealed tube containing the radium which is eventually to be mixed with the sulphide is first measured. The zinc sulphide is placed in a glass cell of known thickness, and the radium placed very close to the face of it, remote from the chamber. Observations are taken with the cell empty and with it full, and from the knowledge of the apparent density of the material in the cell the mass absorption coefficient under the conditions of experiment is calculated. (2) The sulphide is put into a glass tube of known diameter. The radium tube is then placed on the axis of this tube and its radium content again measured. (3) The contents of the radium tube are intimately mixed with a known quantity of the sulphide and placed in a glass tube of known diameter. The values of the radium content before and after mixing give sufficient data to calculate the absorption coefficient of the radiation in the sulphide. (4) The luminous compound made for the third of the above methods is placed in a glass cell of known thickness and the absorption of the radiation from another tube of radium measured in it by the first method.

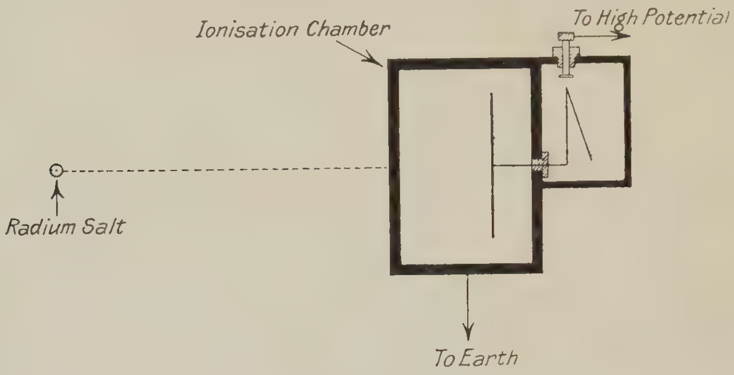

FIG. 1,

ABSORPTION OF γ-RADIATION IN SODA GLASS.

A preliminary investigation was carried out on the absorption of different kinds of soda glass. It was necessary to know this figure in order to correct for the absorption of the radiation in the walls of the glass vessels containing the compounds. A number of determinations of the absorption coefficient (μ) in soda glass, under the conditions of experiment, gave the value 0·105 cm.$^{-1}$.

OBLIQUITY CORRECTION.

In the first of the above methods of measurement the cone of the rays passing into the ionisation chamber, which was 14 cms. in diameter internally, has an appreable angle at the vertex. The semi-angle in the present case varied from about 4 deg. to about 17 deg., and for the larger angles a correction had to be made for the obliquity of the radiation. This was calculated by means of the following formula, due to Soddy and Russell* :—

$$\frac{I}{I_0} = \frac{e^{-\mu t} - \cos\theta\, e^{-\mu t/\cos\theta} + \mu t[Ei(-\mu t)] - \mu t[Ei(-\mu t/\cos\theta)]}{(1-\cos\theta)}$$

where I_0 and I are the intensities of the radiation before and after traversing the salt.

A full curve was drawn, showing the relation between $\frac{I}{I_0}$ and μt, for given values of θ, from which the value of μ could be obtained for any intensity ratio measured.

RESULTS.

Two radium standards were employed in the work, the one (Standard A) containing about 15 mgms. and the other (Standard B) 2·5 mgms. of radium element. The following table shows the variations that were obtained in the values of the absorption coefficient with the two standards when placed at different distances in front of the ionisation chamber :—

TABLE I.

Standard A.		Standard B.	
Semi-angle θ of cone of rays.	μ(cm.$^{-1}$).	Semi-angle θ of cone of rays.	μ(cm.$^{-1}$).
8° 19′	0·100	16° 31′	0·095
7° 27′	0·099	13° 45′	0·101
5° 50′	0·099	11° 46′	0·100
5° 6′	0·101	10° 17′	0·101
4° 31′	0·103	9° 7′	0·100
4° 1′	0·097	8° 12′	0·102

The values of the absorption coefficient given in the table have been corrected for the obliquity of the rays. The mean value of μ is 0·100 cm.$^{-1}$.

Another determination of μ by the same method for a different sample of sulphide gave the value 0·103 cm.$^{-1}$. The average value of the apparent density of the zinc sulphide employed for these determinations was 2·03 grammes per cubic centimetre, so that the mean value of the mass absorption coefficient is 0·050.

A determination of the absorption coefficient, when the radium tube was placed axially inside a wider tube filled with zinc sulphide, gave the value μ=0·098 cm.$^{-1}$.

The radium was now carefully mixed with a known quantity of the sulphide. This was carried out by the dry method and it may be of interest to indicate the procedure adopted. The contents of the radium tube were transferred into an agate mortar ; it was difficult to remove all the salt from the tube as some of it adhered to the walls, even when the tube was gently tapped. The final cleaning was carried out by putting a little zinc sulphide into the tube and rubbing it on the inside walls by means of a small camel-hair brush. The operation was repeated five or six times, using only a small quantity of sulphide in each operation, until all the material clinging to the walls was removed. Having now transferred th radium with the small quantity of zinc sulphide used in the cleaning process in the mortar, it was ground for about 20 minutes into a fine powder. A sma quantity of zinc sulphide was afterwards added, say, about 1 gramme, and mixe with the radium by means of a spatula, taking care not to apply much pressure t the sulphide so as to avoid breaking up the crystals, which should be kept as larg as possible for the best luminous effect. More sulphide was added and agai thoroughly mixed with the salt already in the mortar. If the quantity of zin sulphide is not too large it may all be added little by little to the radium, stirrin constantly and keeping about 2 grammes of sulphide in hand for final washings o camel-hair brush, spatula, and funnel used to transfer the compound from morta to the tube in which it is sealed. It is advisable not to fill the tube, so that the sal may be well shaken for final mixing. In this way a very homogeneous mixtur was obtained.*

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* Soddy and Russell, Phil. Mag., XIX., 725 (1910).

The compound was finally sealed in a glass tube whose internal diameter wa 14·1 mm. and whose thickness of wall was 0·85 mm. The growth of activity of th compound was kept under observation for about five weeks, after which perio it had reached a state of radioactive equilibrium. The maximum value of its activit was found to be equivalent to 2·314 milligrammes radium element, whilst the activit of the radium tube before mixing with the sulphide was equivalent to 2·51 milligrammes. Calculating the value of μ to a first approximation from the simp formula $I=I_0e^{-\mu}$, where t is 7·05 mm., we have, after correcting for the absorptio in the glass, $\mu=0{\cdot}107$ cm.$^{-1}$. There will undoubtedly be a slight loss of radiu during the process of mixing the compound which will tend to increase the value o μ. A loss of 0·5 per cent. of the salt during mixing would in the present case alte the value of μ from 0·100 cm.$^{-1}$ to 0·107cm.$^{-1}$, so that we may say that the abov figure is in fair agreement—allowing for experimental error—with the valu obtained by the other methods.

The fourth method of measurement, in which the luminous compound wa removed from the sealed tube, placed in a glass cell and the absorption of th radiation from another radium tube in it measured, gave 0·100 cm.$^{-1}$ for the valu of μ.

There is, therefore, no appreciable difference between the values of th absorption coefficient of the radiation in zinc sulphide when measured by any on of the four methods described. The two values of μ obtained by Paterson, Wals and Higgins may be explained if it be assumed that the compound when finall measured had not reached a state of radioactive equilibrium. It is possible for radium salt to reach a steady maximum value of activity and yet not be in a stat of true radioactive equilibrium. This may occur if the containing tube is imperfectl sealed, so that there will be a constant small leakage of emanation. The rate o production of emanation by the radium and the rate of leakage of emanation fro the tube will, after a time, reach a steady state giving a fictitious state of equilibrium The maximum value of the activity will in this case be smaller than the value attaine when the salt is in a state of true radioactive equilibrium, so that if the luminou compound were not properly sealed in its enclosure, the value of the absorptio coefficient of the radiation measured by the third of the above methods would b too great.

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* The authors are indebted to Mr. F. H. Glew for the details of this method of preparin luminous compound.

THE RATIO OF THE TRUE TO THE APPARENT RADIUM CONTENT OF TUBES OF LUMINOUS COMPOUND OF DIFFERENT DIAMETER.

The most reliable value of the absorption coefficient of γ-radiation in zinc lphide is that measured by the first of the above methods, which gave the mean lue 0·101 cm.$^{-1}$. Knowing this figure and assuming that the radium is distributed niformly throughout the body of the sulphide, we can calculate the ratio of the ue to the apparent radium content of tubes of different diameter filled with luminous mpound. Suppose the tube containing the compound is cylindrical and of internal dius a cm. If I is the measured intensity of the γ-radiation emitted by e tube and I_0 the intensity if no absorption had taken place, it can readily be own,* if the correction due to the obliquity of the rays be neglected, that

$$\frac{I}{I_0}=\frac{1}{\mu a\sqrt{\pi}}\sum_{r=1}^{r=\infty}(-1)^{r+1}\frac{(2\mu a)^r}{\underline{|r+1}}\,.\,r\,.\,\frac{\Gamma\frac{r}{2}}{\Gamma\frac{r+1}{2}}$$

$$=C_0+C_1(\mu a)+C_2(\mu a)^2+\ \ldots\ldots$$

here μ is the coefficient of absorption of the rays per centimetre thickness of mpound. The values of the constants C_0, C_1, C_2,...are given in the following table :—

TABLE II.

Coefficient.	Value.	Coefficient.	Value.
C_0	+1·000000	C_7	—0·001642
C_1	—0·848826	C_8	+0·000347
C_2	+0·500000	C_9	—0·000067
C_3	—0·226354	C_{10}	+0·000012
C_4	+0·083333	C_{11}	—0·000002
C_5	—0·025869	C_{12}	0·000000
C_6	+0·006944		

Since the value of the absorption coefficient for soda glass is approximately e same as that for zinc sulphide we may, without serious error, calculate by eans of the above series the ratio of the true to the apparent (or measured) adium content for tubes of different *external* diameters on the assumption that =0·101 cm.$^{-1}$. The values calculated in this manner are given in the following able :—

TABLE III.

External Diameter of containing tube in cm.	$\frac{\text{True Content}}{\text{Apparent Content}}$
0·5	1·02
1·0	1·04
1·5	1·07
2·0	1·09
2·5	1·11
3·0	1·13

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* See Paterson, Walsh and Higgins (loc. cit.). There is a misprint in the general term of he series as given in their Paper, but the values of the constants are the same as given here.

ABSORPTION OF γ-RADIATION IN OTHER SALTS.

The absorption of γ-radiation was also measured in a number of other sa special attention being paid to barium salts with which radium is closely associat The measurements were made by the first method outlined above. Each salt powdered in a mortar and its apparent density determined from the weight of t salt required to fill the glass cell in which the absorption was measured. The thic ness of material traversed by the rays was 1·12 cm. The following table contains summary of the results obtained :—

TABLE IV.

Salt.	Apparent Density in gms. per c.c.	Absorption Coefficient cm.$^{-1}$.	Mass Absorption Coefficient.
ZnS	2·03	0·101	0·050
$BaSO_4$	2·41	0·079	0·050
$BaCl_2$	1·58	0·100	0·053
$BaCO_3$	1·88	0·046	0·055
PbO	4·21	0·294	0·070
Bi_2O_3	2·89	0·206	0·071
U_2O_5	3·93	0·315	0·080

The mean value of the mass absorption coefficient in the barium salts may taken as 0·053. By the aid of this value and a knowledge of the apparent densi of a barium salt containing a small percentage of radium, a table similar to Table I above may be compiled, giving the ratio of the true to the apparent radium conte of the salt for tubes of different diameters.

It should be noted that the above values of the absorption coefficients ho when the radiation passes through 0·5 cm. of lead on its way to the chamber. Sin γ-radiation from radium is heterogeneous the values of the absorption coefficien would be different for other thicknesses of lead in front of the ionisation chamber.

DISCUSSION.

Mr. C. C. **PATERSON**: I have been much interested by the explanation which the aut has offered of the divergent results obtained in the previous work to which he has so kin alluded. We were greatly puzzled by this divergence at the time, but I think the suggesti that there was a false equilibrium due to leakage of emanation provides a very probable a satisfactory solution of the question.

. *The Structure of Organic Crystals. By.* Prof. Sir W. H. Bragg, *K.B.E., M.A., F.R.S.*

(Presidential Address, delivered November 11, 1921).

SUMMARY.

For many reasons the structure of crystals of organic substances invites examina- n by the methods of X-ray analysis ; but their molecular complexity would seem to ow great difficulties in the way. It is possible, however, that the difficulties in the se of aromatic compounds may be surmounted by adopting a certain hypothesis, viz., at the benzene or naphthalene ring is an actual structure, having definite size and form, d that it is built as a whole into the organic substances in which it occurs. Reasons n be given why this is *a priori* probable.

The examination of certain organic crystals has been made. The results are in neral agreement with the hypothesis, and lead to various deductions of interest.

HE structure of organic crystals offers a very inviting field of research by the ethods of X-ray analysis. To the organic chemist the relative positions of the oms in the molecule, as also of the molecules in the crystal, are of fundamental portance ; and it is with these relations that the X-rays deal in a manner which new and unique. Moreover, the multiplicity of crystalline forms—and this is ue of both organic and inorganic substances—each so precise and invariable, and obviously related to the atomic and molecular forces, is a sign that if the forces ere better understood it would be possible to account for the forms that are known, d possibly to build others that are unknown. But in order to acquire such a power must learn the crystalline structure, so that the physical characteristics of the hole may in the end be referred to the characteristics of the individual atom. ogress has been made with the examination of the structure of some of the simpler organic crystals ; but organic crystals have been neglected. Their molecular mplexity has been somewhat of a deterrent. Yet, if a way could be found of aking determinations of structure, in spite of the complexity, it seems likely that ey would quickly be fruitful. The substitutions and additions which are so aracteristic of organic chemistry take place in such an immense variety of combina- ons and grades that the slightest knowledge of the underlying mechanism might d to useful comparisons and rules.

I have made a careful study of a few crystals, principally naphthalene and me of the naphthalene derivatives, in order to discover, if possible, some way of ndling the complex molecules. The numerical results will be set out later, and ay, I think, be taken as sufficiently accurate to make foundations for a theory.

I shall endeavour to show that the results can be explained, so far as can be en at present, by supposing the benzene ring or naphthalene double ring to have finite form and size, preserved with little or perhaps no alteration from crystal crystal, and that there are good *a priori* reasons for the supposition. If this inciple be accepted the problem is simplified at once. Naphthalene itself is then be regarded as a structure in which there is but one element, the naphthalene uble ring, and no longer as an aggregate of 10 carbon atoms and eight hydrogen oms of unknown mutual arrangement. A more complex molecule such as either the naphthols is not to be regarded as an addition of one oxygen atom to these , an idea on which nothing can be built, but as a naphthalene double ring of the me size and form as before, except that one particular hydrogen has been replaced by a hydroxyl group. It is then possible to think what changes in the disposit of the molecules might be caused by such a substitution and to compare conceiv solutions with observations on the dimensions of the new crystal. Such a met of procedure is obviously in good agreement with the ideas of organic chemistry

It is convenient to distinguish the facts regarding crystalline structure wh can be obtained by the goniometer and various other means, from the new fa which can be obtained by the use of X-rays. The former are recorded in crystal graphic tables such as are given by von Groth in his " Chemische Krystallographi Naphthalene may be taken as an example. In the fifth volume of von Groth's wo on p. 363, a description of naphthalene is given from which the following data taken :—

MONOCLINIC PRISMATIC.

$$a : b : c = 1{\cdot}3777 : 1 : 1{\cdot}4364\,;$$

$$\beta = 122^\circ\ 49'$$

The monoclinic prismatic class has the highest symmetry of which the mon clinic system is capable, having a diagonal axis and a plane of symmetry perpendicu to it. The figures give the angular, but not the linear, dimensions of the unit c of the structure (Fig. 1) ; the unit cell being the smallest volume, of which; continual repetition without any change in contents or disposition, the whole crys can be formed. In any crystal it is possible to choose the unit cell in many wa but they must all be capable of derivation from one another and must all have t same volume. The angular dimensions are to be considered as including the angu relations to one another of any pair of planes in the crystal, not merely of the pla bounding the cell.

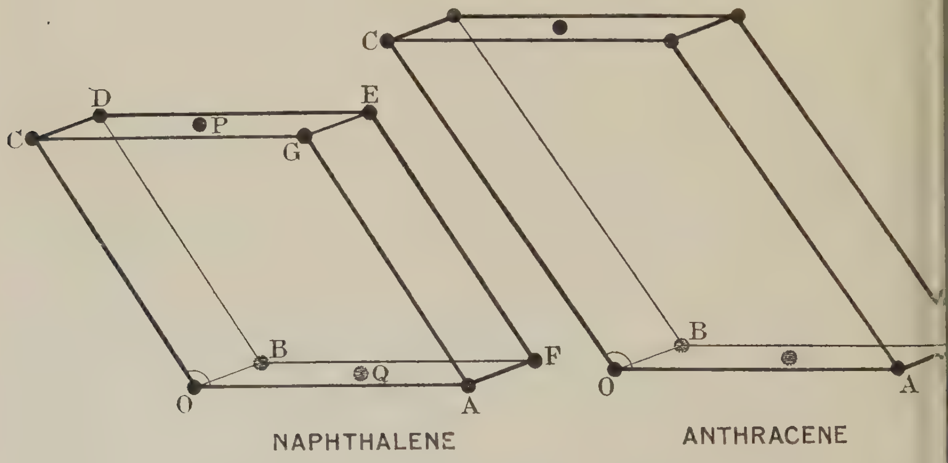

FIG. 1.—UNIT CELLS OF NAPHTHALENE AND ANTHRACENE DRAWN TO THE SAME SCALE.

	$OA=a$	$OB=b$	$OC=c$
Naphthalene	8·34	6·05	8·69
Anthracene	8·7	6·1	11·6

Naphthalene $\alpha = BOC = 90^\circ$, $\beta = COA = 122^\circ\ 49'$, $\gamma = AOB = 90^\circ$
Anthracene $\alpha = BOC = 90^\circ$, $\beta = COA = 124^\circ\ 24'$, $\gamma = AOB = 90^\circ$

It is also stated that the (001) face is the cleavage plane ; and that, in addition, the faces (110), $(20\bar{1})$, $(11\bar{1})$ are found as bounding planes of the crystal. The angles between various pairs of these faces are also given as observed. The specific gravity is stated to be 1·152. Other information is given by von Groth concerning the optical properties of the crystal ; also concerning the methods that have been adopted in growing the crystals from various solutions and the consequent effect on the development of different faces. These facts do not concern us for the moment, but they must be taken into account eventually.

The examination by X-ray analysis gives us the spacings between the planes and, therefore, the linear as well as the angular dimensions of the unit cell. The specific gravity being known, and the actual weight of the molecule, it is possible to find how many molecules are contained in each cell ; generally, two or four. In the case of naphthalene, it is found that, assuming the angular dimensions to be correctly given by the crystallographers, the linear dimensions are :—

$$a=8{\cdot}34,\ b=6{\cdot}05,\ c=8{\cdot}69.$$

These figures are obtained in the following way :—

The actual length of the b axis being represented by b, the mass contained in the cell is

$$b^3 \times 1{\cdot}3777 \times 1{\cdot}4364 \times \sin 122^\circ\, 49' \times 1{\cdot}152 \text{ A.U.}$$

(It is convenient in this work to extend the Angstrom system of units so that an A.U. of area is 10^{-16} cm.^{2}, of volume 10^{-24} cm.^{3}, and of mass 10^{-24} gr.)

The mass of the hydrogen atom being 1·662 A.U., the mass of the molecule $C_{10}H_8$ is $128 \times 1{\cdot}662 = 213$ A.U. Now, from the full results of the X-ray measurements which will be given presently, it is perfectly clear that there are two molecules in each unit cell. Hence, the value b is readily calculated, and the values of a and c also.

Besides these determinations of length, the X-ray method gives also the angle between any pair of planes, whether they form faces or not, provided that a measurable reflection can be found. Also, the relative intensities of the reflection by different faces, as well as the relative intensities of the spectra of different orders given by any one set of planes, yield information as to the distribution of the scattering centres and of the atoms which contain them.

There are two distinct methods of using the X-rays. In the one, which has been used from the beginning, a single crystal is employed. It need not usually weigh more than 2 or 3 milligrammes, and, in fact, it is more convenient that it should be small, since the pencil of reflected rays is then conveniently limited by the size of the crystal without the need for slits, which are still used, however, but as stops only. This method is at present the more precise of the two. Also it permits the measurement of the angles between reflected planes, a knowledge of which is often very useful for identifying the planes.

The second method, first used for crystal analysis independently by Debye, and by Hull, can be used when the crystal is in powder, and can, therefore, be employed when no single crystal can be obtained of sufficient size to measure by the first method. All the spectra of the different planes are thrown together on the same diagram or photograph, and must be disentangled. This is not so difficult as it might seem, because there are not more than one or two lines in each spectrum, and there is generally independent evidence as to the angular dimensions of the

crystal. The spectra of copper, and of some organic substances, shown in Fig
2, 3, 4 and 5, were obtained in this way. The X-ray bulb employed was of the for

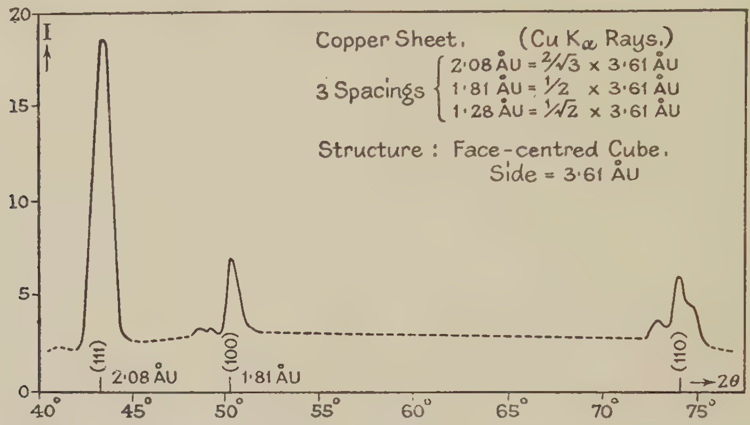

Fig. 2.—Copper Spectrum, Obtained by Powder-Ionisation Method.

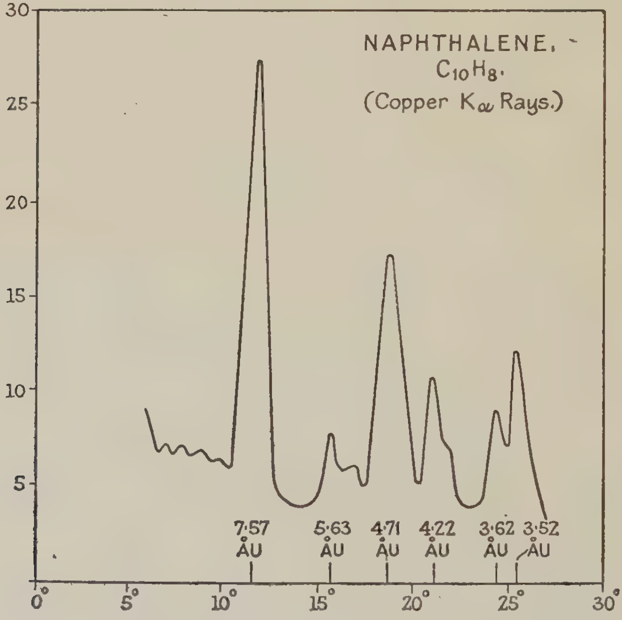

Fig. 3.—Spectrum of Naphthalene—Powder Method. The results agree well wit
those obtained by the Single Crystal Method, but are slightly larger.

described by Müller.* The way of using the powder in conjunction with an ionisation spectrometer has been described in the Proceedings of this Society.† Müller' ulb is very well suited to this method. The spectrometer slit can be brought very ose to the radiator ; so that the slit becomes the source of a strong beam of rays hich is sufficiently divergent to cover a large surface of the plate on which the

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* Phil. Mag., p. 419, September (1921).

† Proc. Phys. Soc., Vol. XXXIII., Part 4, p. 222, June (1921).

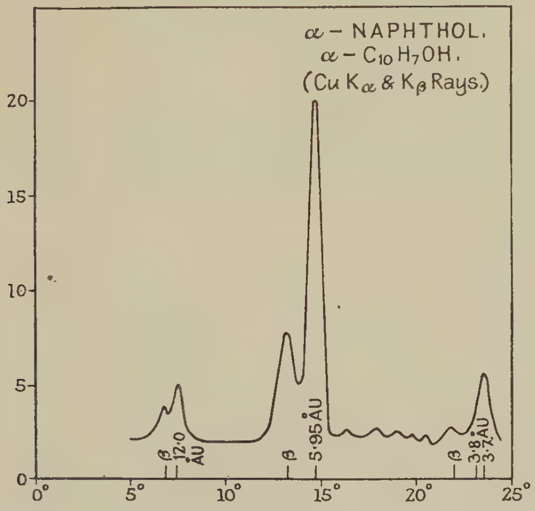

FIG. 4.—SPECTRUM OF α-NAPHTHOL—POWDER METHOD.

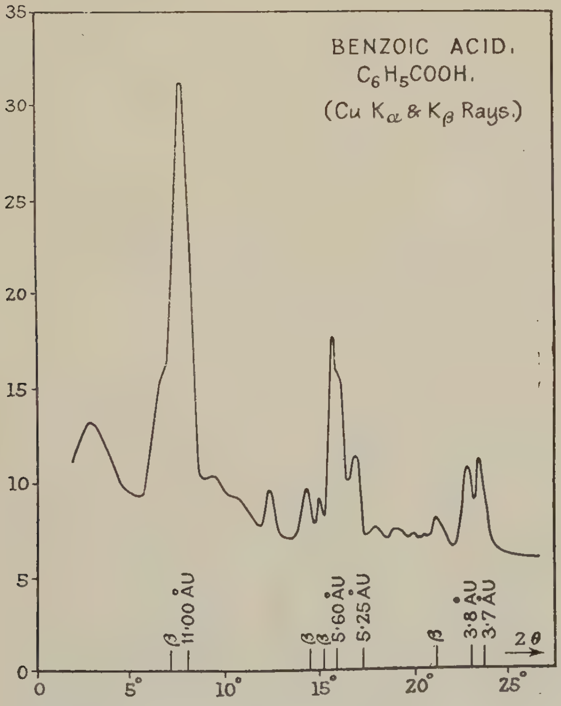

FIG. 5.—SPECTRUM OF BENZOIC ACID—POWDER METHOD.

powder is spread. By a known focussing action (X-rays and Crystal Structure W. H. and W. L. Bragg, p. 31) the rays are converged upon the slit of the ionisation chamber. The anticathode used in these experiments was made of copper ; the long wave lengths of the *K* series of copper give suitable angles of deflection even for the wide spacings that are found in organic crystals. The curve in Fig. 2 shows the spectrum of copper obtained by the use of copper-rays, and gives at once all the necessary information from which the structure of the copper crystal is derived. The piece of copper employed was of ordinary sheet, not prepared in any way. The spectra of the organic substances show how very diversified they are, and illustrate the power of a method of analysis which promises to be of great use, since every crystal has its own characteristic spectrum. The bulb was driven by a transformer

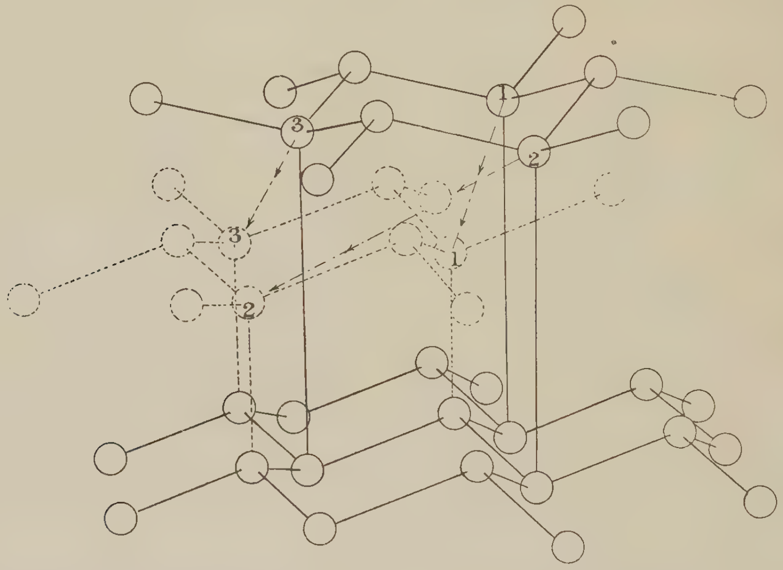

Fig. 6.—The Fine Lines of the Diagram show the Structure of Graphite. By Moving the Top Layer to the position shown by the Broken Lines the Diamond Structure is obtained.

capable of developing half a kilowatt ; but not working to half of its full capacity. Each determination of a point on the curve took about 10 to 30 seconds.

We may now consider what reasons can be put forward for assuming the concrete existence of the benzene and naphthalene rings. If we examine the structure of the diamond we find that the atoms of carbon are tied together so that each is at the centre of gravity of four others. The distance from centre to centre is 1·54 A.U. As I have already pointed out* the rigidity of the diamond and the open character of its structure, imply that great force is required to alter the orientation of any coupling with respect to the other three belonging to the same atom. Were it otherwise, all atoms would seek to be surrounded by as many neighbours as possible ; the substance would be close packed, and its density would be more than double what it is. The structure of the diamond may also be looked on as consisting of a series of puckered ers parallel to a given tetrahedral plane (*see* Fig. 6). A sharp blow may cleave the mond, along one of these layers. If we take a model showing two layers as in phite, lay hold of the upper layer, and move it to the new position shown in the ure, the structure is now that of diamond. I am following Hull's determination;* bye and Scherrer† would flatten out the puckers in the planes of graphite. e point is perhaps not important for our present purpose, but it is necessary, for criptive purposes at least, to choose one form. According to Hull's measure- ents, the shortest distance between each pair of atoms lying in the same layer is rtened from 1·54 A.U.—the diamond spacing—to 1·50 A.U. The distance ween two successive layers has been increased by 1·35 A.U. A carbon atom in e layer is now at equal distances from its three nearest neighbours in the next er, the distance being 3·25 A.U.

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* Proc. Phys. Soc., Vol. XXXIII., Part 5, August 15 (1921).

The bonds between one layer and the next are now greatly weakened; the ostance cleaves readily in thin flakes. One layer slides with great ease over the her, though the bonds between the atoms in any layer are at least as strong as fore. When all the bonds were of the strong kind, the substance, as diamond, s the hardest thing known. When the one set of bonds has been weakened, the ostance, as graphite, is used as a lubricant. Probably its efficiency as such depends th on the weakness of the one set of bonds and the strength of the other. Yet ese new bonds are perfectly definite, and the distance between two layers and, erefore, the 3·25 distance between atomic centres is a perfectly constant and terminate quantity. It should be of great interest to compare the physical con- nts of graphite along and perpendicular to the axis, since in the two cases the two nds of bonds are separately involved. Some of the comparisons would be difficult, t the thermal expansions can probably be compared by X-ray methods.

If the strong bonds between the atoms in a layer remain, and are even drawn ittle tighter when the graphite form replaces that of diamond, it seems very reasonle to suppose that the single ring or multiple rings which are so clearly to be stinguished in the network may be separated out as such without loosening the bonds tween their component atoms. In fact, these latter bonds might be expected to ghten even a little more. Let us assume that a single ring is a benzene ring, a double ng a naphthalene ring, and so on. Taking the spacings as given by Hull for graphite, e dimensions of the benzene and naphthalene frameworks are as shown in Fig. 7. ne figure is constructed to show the arrangement of atoms in the naphthalene ystal, but it will also serve to illustrate the point under discussion. The carbon oms *A* to *F* form a benzene ring, those from *A* to *J* the double ring of naphthalene.

The atom centres *A*, and *G*, are 0·71 A.U. above, and the centres *D* and *J* ne same distance below the plane of the diagram which is supposed to contain all the maining centres. It should be observed that circles are used to represent the atoms s a convenient method of designation, not as implying that the radius (1·50) may *ways* be used in calculating the distance between the centre of any one atom, nd the centre of any other atom.

We may now go on to consider individual crystals; and we take naphthalene rst. It might have seemed more natural to attack the benzene crystal before the aphthalene; but the latter was chosen because it is a very well-shaped crystal, nd is solid at ordinary temperature. Benzene can only be examined under special

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* Physical Review, X., p. 692, December (1917).

† Phys. Zeit., 13, p. 297 (1917).

temperature conditions and then only, with convenience, as a mass of small crysta
The study of benzene and some of its derivatives has been begun, but the greate
attention has been given to the naphthalene crystals, and I will describe now t
results that have been obtained in their case.

In the second column of Table I. are given the angles between certain pairs
planes as calculated from the data already given, viz. :—

$$a=8{\cdot}34,\ b=6{\cdot}05,\ c=8{\cdot}69,\ \beta=122°49',$$

and in the third the corresponding values as observed.

TABLE I.

Angles between.	Calculated.	Found.
100 : 001	57°11′	57°16′
110 : $11\bar{1}$	36°32′	36°34′
$20\bar{1}$: 001	85°45′	85°48′
021 : 001	67°47′	67°35′
$20\bar{1}$: $10\bar{1}$	30°21′	30°23′
210 : $21\bar{1}$	24°12′	24°24′

These figures are quoted in order to show that the X-ray method gives angula
measurements with sufficient accuracy. Indeed the agreement between the value
in the two columns is as good as is usually reported by different observers who hav
used the goniometer (*see* the descriptions recorded by von Groth); and the X-ra
method could really be relied on to a minute of arc if the necessary care and time wer
justified. In this case no special pains were taken.

In the next table are set out the results of the linear measurements. They ar
compared with calculations based on the assignment of *one* molecule to each cel
which is equivalent to supposing each corner of the cell to represent a molecule. Th
object of making this provisional assumption is to show how the position of th
second molecule can be found by comparing the observed with the calculated result

TABLE II.

Plane.	Calculated spacing.	Observed spacing.	Nature of reflection.
100	7·00	3·46	Strong: indication of spacing 6·9
010	6·05	2·95	Very weak.
001 (Cleavage)	7·30	7·30	Very strong: also higher orders.
110 (Natural)	4·59	4·55	Strong.
$11\bar{1}$ (Natural)	4·70	4·63	Moderate.
$20\bar{1}$ (Natural)	4·17	4·12	Strong.
021	2·79	2·76	Very weak.
$10\bar{1}$	7·51	3·71	Very weak.
210	3·04	2·99	Strong.
$21\bar{1}$	3·44	3·39	Strong.

No reflection obtained from following planes :—

011, 012, $10\bar{2}$, 101, 111, $22\bar{1}$, 112, $11\bar{2}$, 221, $21\bar{2}$, 211, 212.

The table shows that the 100 and 010 spacings are only half what they shoul
be if there were molecules at cell corners only. (N.B.—Only one-eighth of a corne
molecule is within the cell, and the whole eight count for one whole molecule withi

cell.) But the 001 spacing is right. We conclude that there is a molecule at h of the points P and Q (*see* Fig. 1), each contributing half a molecule to the l: and that these are in all respects similar to the corner molecules. Molecules ced at P and Q interleave the planes 100, 010, and also $10\bar{1}$ by other planes of ual density which halve the corresponding spacings. The planes 110, $20\bar{1}$, 021, are affected because they already contain P and Q.

It should be mentioned, however, that the 100 plane seemed to give a small ectrum at half the angle which gave the principal reflection ; this would indicate at the second molecule was not quite similar, in orientation or some other particular, the first, or was not exactly at P and Q. This is also suggested by the fact that 0 and $2\bar{1}\bar{1}$ give the calculated spacings, whereas half values might be expected.

Although so much information is given in these tables, some of which we have ed and some of which we cannot fully use for lack of knowledge, yet it would be peless to try to arrange the eighteen atoms of naphthalene on the basis of what s been learnt, without some helpful hypothesis. But we now take the naph- alene double ring as described. Its dimensions are such that it seems quite ssible to fit two of them into the cell, if we had some indication as to their orientation ereto.

As to this we get a strong hint from a comparison of naphthalene with anthra- ne ($C_{14}H_{10}$), whose construction shows three rings in a line, as against the two naphthalene. The crystallographic data of the latter are :—

$$a : b : c = 1{\cdot}4220 : 1 : 1{\cdot}8781, \quad \beta = 124^0\ 24'.$$

Specific gravity$=1{\cdot}15$.

The crystals themselves are very small flakes, and it was not possible to find e which could be conveniently treated by the single crystal method. However, pressing a number of them together against a flat disc, so that all the 001 planes re parallel thereto, however oriented they might be otherwise, it was easy to t a sufficiently accurate determination of the 001 spacing, and therefore the linear mensions of the unit cell. It appeared that there were two molecules to the cell, for naphthalene. The dimensions were :—

$$a=8{\cdot}7, \quad b=6{\cdot}1, \quad c=11{\cdot}6.$$

If these dimensions are compared with those of naphthalene (*see* Fig. 1), it will seen that while a, b, and β remain nearly the same, the c axis has lengthened nsiderably, the difference amounting to 2·9 A.U. nearly. Now the extra ring, of the benzene dimensions, should be responsible for an addition of 2·5 A.U. nearly the molecule.

It is reasonable to conclude that the molecules in both crystals lie end to end ong the c axis, and that the structures are similar.

The over-all lengths of the two molecules, without allowance for the hydrogen oms at their ends, that is to say, in the β-positions, are 6·41 and 8·86 respectively. here is, therefore, a vacant space between the ends of two molecules of rather more an 2 A.U., into which two hydrogen atoms have to be fitted. This agrees very ell with what might be expected ; only it must be remembered that we have no efinite indication from studies of crystal structure as to the actual distance between e centres of a carbon and a hydrogen when united by a valency bond, nor between vo hydrogen atoms not so united.

We have still to decide in what plane, passing through the *c* axis, the molec is to be placed, and we have less clear indications with respect to this point th those that have guided us hitherto. On making up a model, however, it is se that it is much more likely that the plane of the molecule lies nearer to the *ac* pla than the *bc* plane. The molecules lock together much better if that is so. Mor over, if the molecules lie in the *bc* plane, they would be close neighbours in th plane, and at the same time there would be wide gaps between consecutive plane The plane 100 should, therefore, be prominent, most probably a natural face, perha even a cleavage plane : whereas it is neither of these things. But if the molecules l in the 010 plane the form of the crystal seems much easier to understand, as w shall see later. The β-hydrogens of each molecule lie up against the correspondin hydrogens of the next and the 001 plane passes through them all. It would appe to be the weakest junction in the crystal, and therefore the 001 plane is the cleavag plane.

It must be observed that in the junctions between molecule and molecule the are forces far weaker than the valency forces, which latter unite the atoms of th same molecule. It is the former which bind the molecules into the crystal, nevertheless.

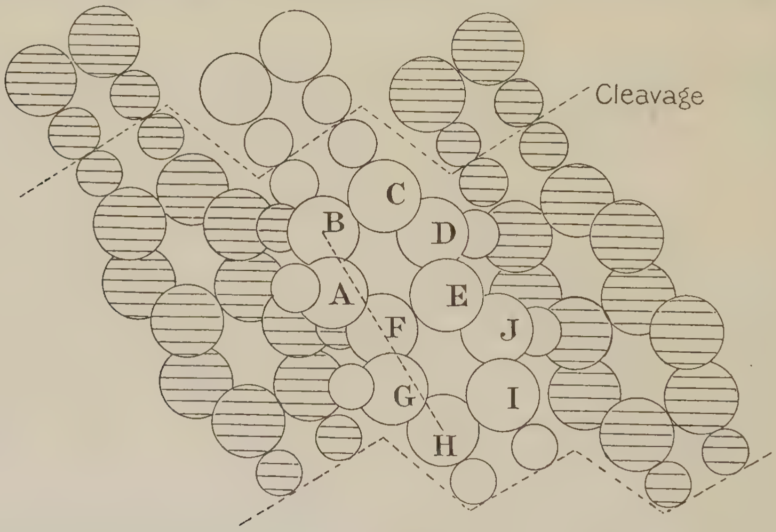


Fig. 7.—Showing Mutual Relations of Three Naphthalene Molecules and Parts o Others.

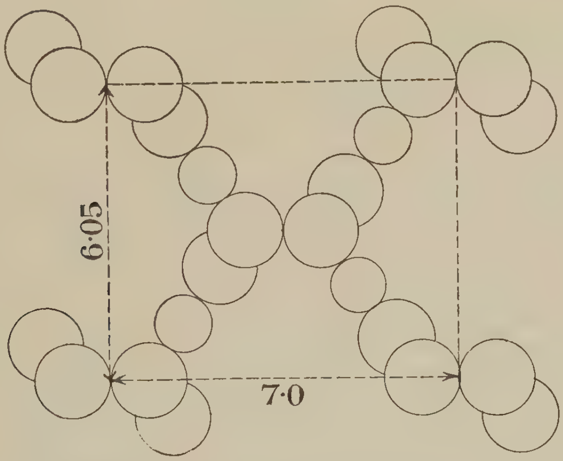
The unshaded circles between the two cleavage planes represent a molecule as at *Q* (Fig. 1 The shaded represent molecules *B* and *F* in the same figure. The small circles represent hydroge atoms, but their size is uncertain.

Diameter of carbon atom=1·50. *BH*=4·92. Projection of *AD* on the plane of th diagram=2·50.

When the model is put together in the way now indicated it is found that a the α-hydrogens, those that are atached to the sides of the molecule, lie up agains carbon atoms of the next molecule, and that there is an appropriate space waitin for each, of magnitude about 1 A.U., the actual value depending on the orientatio molecule. The forces exerted at these junctions, though far weaker than cy forces, are stronger than the forces between two β-hydrogens ; and therefore crystal is ruptured it is the latter which give way first. The forces exerted e α-hydrogens do so across the 110 and 1$\bar{1}$0 planes, and it is not surprising d that the latter make natural faces of the crystal and give a strong reflection. tructure now found is a very empty one : it is like lace-work in space. That be expected, since the specific gravity is so low. The structure is shown in 7 and 7A.

Ve may now attempt to find the structure of a naphthalene molecule in which complexity has been introduced by a substitution. We take acenaphthene. . B. Cohen was kind enough to give me some good crystals weighing each a milligrams.

n this case the molecule has been made lop-sided by the substitution of a group o carbon and four hydrogen atoms for the two hydrogens on one side. It is a



7A.—SECTION OF NAPHTHALENE CELL PERPENDICULAR TO THE AXIS OF *c*, SHOWING α-HYDROGENS CONNECTING THE MOLECULES SIDE TO SIDE.

ing fact that the crystal has now gained in symmetry ; the unit cell is rectangular. crystallographic data are :—

Orthorhombic (with the fullest symmetry of that class)—

$$a : b : c = 0{\cdot}5903 : 1 : 0{\cdot}5161.$$

pears that there are four molecules to the cell, and that

$$a = 8{\cdot}32, \quad b = 14{\cdot}15, \quad c = 7{\cdot}26 \quad (\textit{see}\ \text{Fig. 8.})$$

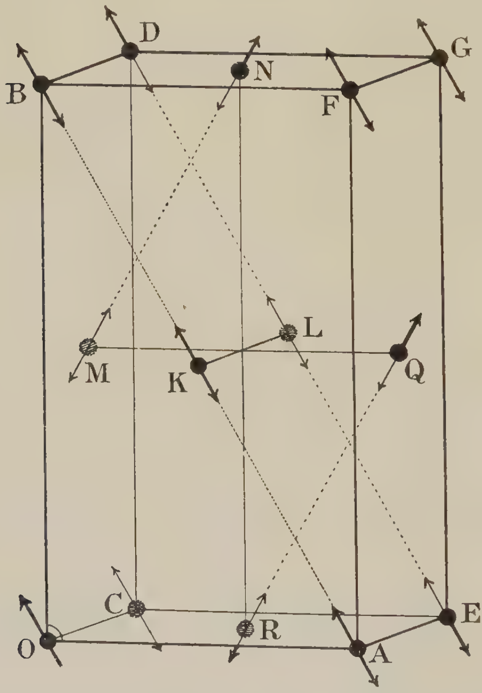
doubling of the number of molecules, as compared with naphthalene, suggests symmetry has been obtained by an arrangement in which two of the unmetric molecules are the images of the other two across one of the principal es, *ab*, *ac*, or *bc*.

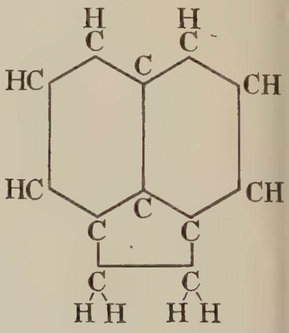
s the symmetry is so high, it is natural to test first the hypothesis that there is olecule at each corner of the unit cell, and one in the middle of each face, which arrangement will account for four molecules. The calculated spacings in the foll ing table are in accordance with this supposition.

TABLE III.

Plane.	Calculated spacing.	Observed spacing.	Remarks.
100 (Natural)	4·16	4·16	Moderate stren
010 (Cleavage)	7·07	7·1*	—
001	3·63	3·63	Moderate stren
110 (Natural)	3·57	7·18	Moderate stren
101	2·74	2·74†	Very weak
011	3·23	3·19	Very weak
111 (Natural)	5·10	5·1	Weak

There is complete agreement, except for the 110 plane. Assuming that th will be an explanation of this as we proceed further, we now look where a mole





ACENAPHTHENE

FIG. 8.—UNIT CELL OF ACENAPHTHENE, CONTAINING FOUR MOLECULES.
$OA=8{\cdot}32$, $OB=14{\cdot}15$, $OC=7{\cdot}26$.

of length about 8·69 (as in naphthalene) may be placed in position, and observe t $KB=KF=\sqrt{(4{\cdot}16^2+7{\cdot}07^2)}=8{\cdot}23$, and these two are images of each other in 100 plane. This is not quite long enough. Let us, however, suppose provision that the molecules at the corners of the cell and at K and L lie parallel to AB, with their planes coincident with (001). Let those at $MNQR$, which between th contribute to the cell the other two molecules that belong to it, lie in the para e, but slope the other way, parallel to *MN* or *QR*. If this is so, we see why 10 plane *ABDE* in the figure seems to have a double spacing ; it is because the sets of molecules—viz., such as *A*, *B*, *D*, *E* of the one set, and *M*, *N*, *Q*, *R* of ther, are very differently disposed towards this plane. One set lies parallel to it, ther not. Consequently the spacing is twice what it would have been had the cules been all alike with respect to this plane, and is equal to the perpendicular *F* or *G* upon the plane *ABDE*. This is true of the 110 plane alone, out of those ut in the table above.

---

* Not observed directly, but inferred from other observations.
† There is an indication that alternate planes differ somewhat.

Once more the cleavage plane (*e.g.*, *BDGF*) passes through the junctions of drogens. On constructing a model it is found that the projecting addition on the of a molecule seems to engage with a neighbouring molecule on that side of the r on which its own addition does not lie. The crystal is thus laced together ;

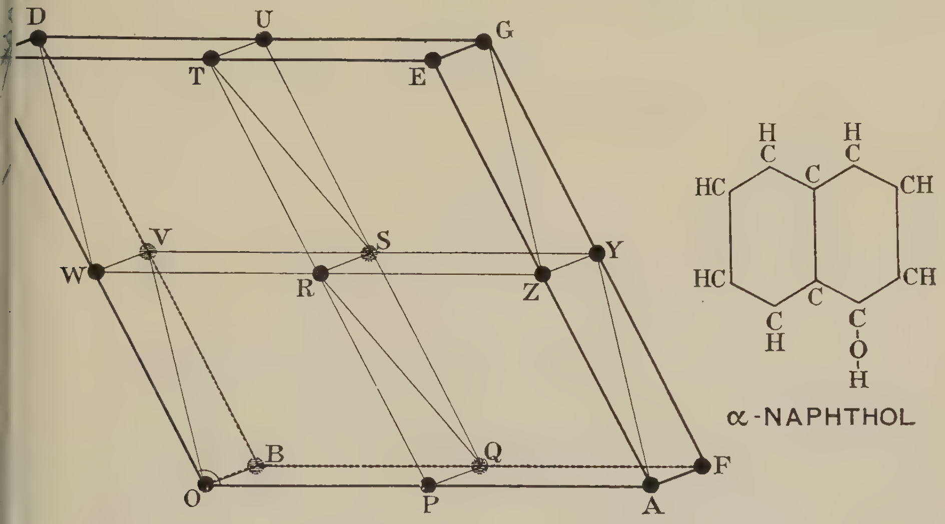


Fig. 9.—Unit Cell of α-Naphthol Containing Four Molecules.
$OA = 13{\cdot}1$, $OB = 4{\cdot}9$, $OC = 13{\cdot}4$, $\beta = 117^\circ\ 10'$.

in lines which for alternate layers parallel to the cleavage plane (010) are parallel *E* and *AC* respectively.

The two points left doubtful are the small difference between the length of the ecule in this crystal and in naphthalene, and the fact noted with respect to the ne 101 (*see* the table).

We now take *α*-naphthol. Here one of the hydrogens at the side of the molecule been replaced by *OH*. It was difficult to obtain without special preparation rystal large enough for the single crystal method ; but on sorting over a considerable amount of material a few flakes were found, each two or three millimetres oss and a fraction of a millimetre thick. With one of these some very good observations were made, as recorded in the table below. The crystallographic data are :—

Monoclinic prismatic.

$$a : b : c = 2{\cdot}7483 : 1 : 2{\cdot}7715, \quad \beta = 117^\circ\ 10'.$$

Specific gravity $= 1{\cdot}224$.

s natural to suppose that the similarity of these figures to naphthalene, when the

a and *c* have been halved, means that the naphthalene cell has been increase
times in order to allow all the four *a* positions to be represented in due order.
it turns out that the crystal is of a totally different structure. There are four mole
in the cell (*see* below), as defined in accordance with the above data. If the
thalene cell had been repeated four times, there would have been eight molecu
the larger cell.

The linear dimensions are :—

a=13·1, b=4·9, c=13·4 (*see* Fig. 9)

and it is clear that the molecules are placed as in the figure, one at each point indic
by a letter. This is, perhaps, seen most readily if the calculated figures of the follo
table are reckoned as if there were one molecule at each corner of the large cell o
viz., *OAFBCEGD*. We then have the following comparison :—

TABLE IV.

Plane.	Calculated spacing.	Observed spacing.	Remarks.
Cleavage 001	11·92	12·00 *	Strong
002	5·96	5·95*	Very strong
210	3·75	3·76	Weak
$21\bar{2}$	3·69	3·68	Rather weak
100	11·82	...	...
200...	5·91	5·84	...
$10\bar{1}$	11·38	...	...
$20\bar{2}$	5·69	5·62	†

Also angles between 210, $21\bar{2}$ 36° 24′ (calc.) 36° 16′ (obs.)
and between $10\bar{1}$, 100 57° 48′ ,, 58° 5′ ,,

In the cases where a bracket occurs in this table, it is meant that a refle
shows the existence of a certain spacing between planes, while a very strong refle
at twice this glancing angle shows that this spacing is halved by other planes diff
considerably from the first. See, for example, Fig. 4, where a small peak show
001 spacing 12·00, and a much larger peak at twice the angle corresponds to
the spacing. This agrees with a supposition that there are molecules at every p
indicated by a letter, but those at *VSYZRW* are very different in some way fron
others.

The reflection of 100 in the powder photograph would be indistinguishable
001, and perhaps also the $10\bar{1}$. The powder method does not give very clear inc
tions, therefore, of the existence of the spacings 11·82 and 11·38, because all th
observed may be due to the 11·92, which must be strong, as 001 is the cleavage pl
The single crystal method gave the proper reflection of the 200 and $20\bar{2}$ plane
shown in the table ; but the reflections belonging to the double spacings 100 and
were not looked for, and probably would have been very hard to find with so s
a crystal. However, the combined data are amply sufficient for the conclusion
the four cells in the figure really imply four different orientations of the molec
and that if we place molecules exactly at the corners of the large fourfold cell
other molecules are very nearly at all the other corners.

---

* Powder method only.

† The symbol 002 is used to denote the second order reflection of 001. When it is abnorm
strong, it follows that the 001 planes are interleaved.

We have this further indication that, though the half of the c axis, $OW=6{\cdot}7$, is short to take in the naphthalene molecule, yet the distance OV is equal to 8·3, actly the distance into which the molecule seemed to fit in acenaphthene. Once ain, then, it would seem that these lopsided molecules lie criss-cross, as represented the lines parallel to OV in the figure ; but they now lie edgeways on top of one other, not flatways, as in the other crystal. It is the "*a*" axis that now runs along line of crossings, not the "*c*" axis. The cleavage plane again passes through the unctions.

When a model is made, it is found that the hydroxyl groups fit into their places y naturally, and that if they link together the tops of the molecules in any one er parallel to the 001 plane, they link in the same way the bottom ends in the xt layer. This brings pairs of hydroxyl groups rather close together, as one might pect ; and it looks as if the attraction existed specially between the two oxygens. e attraction is exerted across every alternate 001 plane. The other 001 planes the cleavage planes. All the other linkings appear to be done by α-hydrogens.

The crystallographic data of β-naphthol are :—

$$a : b : c = 1{\cdot}3662 : 1 : 2{\cdot}0300 \; ; \; \beta = 119^\circ 48'$$

Specific gravity=1·217.

This has been examined by the powder method only, as a single crystal was not dily obtainable. There is a very strong reflection for the spacing 7·90. This sufficient to fix the linear dimensions of the cell. If we put

$$a=5{\cdot}85,\; b=4{\cdot}28,\; c=8{\cdot}7 \quad (\textit{see}\ \text{Fig. 10})$$

weight of material in the cell is equal to that of one molecule, and the 001 spacing 7·76 in close agreement with what is found.

Comparing these figures with those for the smaller cell of α-naphthol such as $QPWVSR$ (Fig. 9) which are 6·55, 4·9 and 6·7 respectively, it is clear that the noval of the OH group from the side to the end of the molecule has caused the cell shrink sideways in both a and b directions and to grow along the c axis ; this nfirms the hypothesis that the molecules in these crystals lie mainly along the xis. The cleavage plane still cuts across the β-junctions.

The question naturally arises as to whether this is really the unit cell. Ought not to be multiplied by four, as for α-naphthol, because there are four β-positions, well as four α-positions ? The evidence in favour of the extension seems very ong. In the first place, it would be the only case, if the present crystallographic ta were correct, in which a cell contained one molecule only. Moreover, if all the lecules were oriented alike, their dissymmetry would lower the symmetry of the ystal. The symmetry which is assigned to it can only be obtained by making unit cell large enough to contain four molecules so that all the β-positions may represented. Also the hydroxyl groups will surely tend to draw together and use pairs of molecules to point opposite ways. For these reasons alone it seems ely that the structure is really the same as that of α-naphthol. The length OV ich was 8·3 for α naphthol becomes 9·7, a growth of 1·4 which seems consistent th the value of the diameter of the oxygen atom, viz., 1·30.*

No other naphthalene derivatives have yet been examined by the X-ray methods, t there are, of course, large numbers that may be expected to give useful results.

* W. L. Bragg, Phil. Mag., p. 169, Aug., (1920).

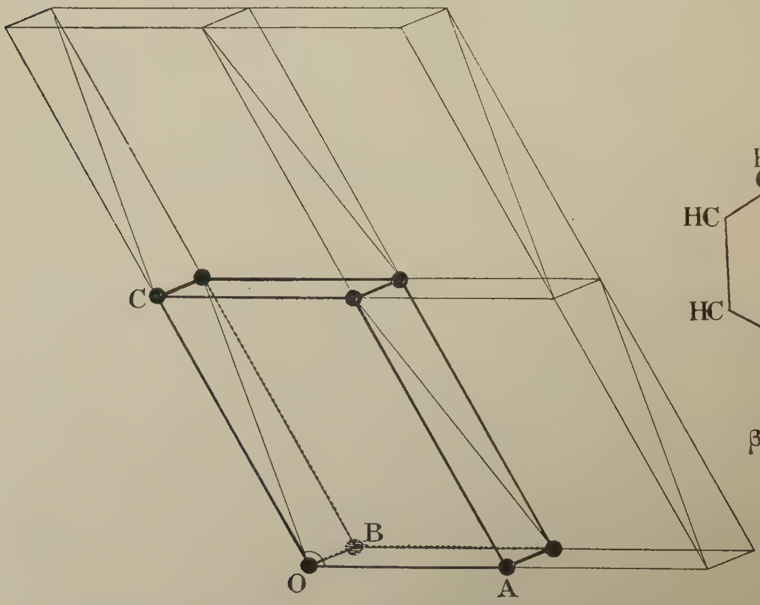
It is worth remarking that the unit cell of α-naphthylamine has almost exactly same dimensions as that of acenaphthene, if it be supposed to contain four molecu Both are rectangular ; the acenaphthene dimensions are :—

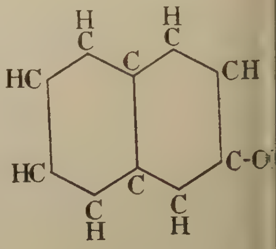
$$a=8{\cdot}32,\ b=14{\cdot}15,\ c=7{\cdot}26,$$

those of α-naphthylamine are :—

$$a=8{\cdot}62,\ b=14{\cdot}08,\ c=7{\cdot}04,$$

provided that the *a* and *b* axes of the crystallographers are interchanged in t latter crystal. Their habits also correspond closely, and although the latter cryst has no clear cleavage plane it has a very prominent (010) face which may represe the 010 cleavage of the former. Assuming that the arrangement is therefore t same in the two crystals, the length of the molecule in the new case is $\sqrt{}(4{\cdot}31^2+7{\cdot}0$ $=8{\cdot}25$; as against 8·23 for acenaphthene and 8·31 for α-naphthol. In all the three cases the lopsided lines of molecules are laid criss-cross upon one anoth





β-NAPHTHOL

FIG. 10.—UNIT CELL OF β-NAPHTHOL, IN THICK LINE, CONTAINING ONE MOLECULE. $OA=5{\cdot}85,\ OB=4{\cdot}28,\ OC=8{\cdot}7,\ \beta=119°\ 48'$.

whereas in naphthalene the lines are parallel to each other. The somewhat great length of the molecule in naphthalene, viz., 8·7, may be connected with the differen in arrangement.

I have made some measurements of benzene compounds, but the analysis of t results is incomplete ; and until benzene itself is examined data are wanting th must be important. It is worth mentioning, however, that benzoic acid has a ve wide spacing between the (001) planes. The dimensions of the unit cell containi four molecules, are :—

$$a=5{\cdot}44,\ b=5{\cdot}18,\ c=21{\cdot}8,\ \beta=97°05'.$$

Experiment by the powder method shows that there is a very strong 10·9 spacin implying that the sheets of molecules lie at this distance apart parallel to the 001 plane. If the crystallographic data are correct these sheets differ alternately. Even a 10·9 spacing is very wide, and this suggests that extended *COOH* groups are required to bridge across the intervening space. Such a bridge may be made, of course, by a *CO* extension from one ring joining on to an *OH* extension from another ring. These bridges part very easily, it would appear ; the crystal flakes at the least touch.

The work that I have now described may conveniently be divided into two parts. The first is experimental, and shows, I think, that a large mass of new and valuable information respecting the linear dimensions of organic crystals is to be gained by the methods of X-ray analysis. In the second, I have ventured to suggest and apply a certain principle, viz., that the benzene and naphthalene rings as well as other ring combinations have actual form and dimensions which are nearly, if not quite, the same when they are built into different compounds. Such a principle requires much more illustration before it can be finally established ; for the present, however, it shows promise of providing an entry into a very wide field which otherwise seems difficult to break into.

Moreover, the idea fits in extremely well with the work of Langmuir on surface films, and with such results as those described by Adam,* and with the ideas and experimental results of several other workers.† We see clearly that the forces that bind atoms together are of more than one kind. The very strong valency bonds, whether explained as due to electron sharing, or in any other way, are exemplified by all the linkings in diamond and by the linkings in the planes of the graphite flakes. But, besides these are other bonds of much weaker character, such as those extending between an atom in one graphite sheet and its three nearest neighbours in the next. Such bonds as these unite the molecules of the organic compound so as to form the crystal. They are of varying degrees of strength ; the cleavage plane shows where they are weakest. They are definitely associated with special points on the molecule as we see from the facts of crystallisation. When a crystal forms in a liquid, or by sublimation, the molecule that attaches itself correctly, and with proper orientation to others already in position, is the one that stays there and resists the tendencies of other drifting and thermally-agitated molecules to remove it. It is fixed by the attachment of certain points on its own structure to certain points on the structure of the other molecules. The beautiful exactness of crystal structure is evidence of the precision with which this adjustment is made ; and at the same time of the definite molecular form without which precision would be impossible.

The molecules of naphthalene arrange themselves side by side, the α-hydrogens of each molecule seeking to attach themselves to the carbon atoms of its neighbours ; valency bonds do not enter into this. Whether there is some special tendency to make an attachment with a carbon atom in the centre of the molecule as seems to be the case, or whether the general slant of the molecules is due to some other cause, is a matter for further inquiry. But it is the side-to-side combination that is preferred in comparison with the end-to-end ; the former gives up more energy than the latter. Consequently the crystal grows out quickly into thin sheets. We may further imagine that sometimes, and more often in certain crystals than in others, as well as more often with respect to one plane in a given crystal than to other planes, the geometry may permit a molecule to attach its active points to those of other mol cules, already settled, in more than one way which does not carry on the regular of previous attachments. The crystal starts a new growth, and twinning resul

---

* Proc. Roy. Soc., XCIX., July (1921.)

† *See* a list given by Adam, loc. cit. p. 351.

The surface films studied by Langmuir and by Adam and others were n formed of aromatic compounds such as the few that have been investigated crystals. But the strength of the side-to-side attachment of their molecules compared with the end-on attachments seems to be exactly the same in effect ar to lead to similar results. In a mass of oleic acid the long molecules are lying in a ways, and their heat motions do not give them a chance of attaching themselv at sufficient points at the same instant. But when the hydroxyl ends of some them root themselves in the water, these molecules are held quiet for a momen and the others quickly slip into their places and link on side to side. So the fil spreads out quickly over the water. It is a kind of catalytic action ; the water do not itself draw the film flat, but helps the molecules of the film to arrange themselve In a sense, the film has negative surface energy.

The arrangement of molecules in crystals or in surface films or interfaces betwee liquids and solids cannot be fully explained as due to forces which are mere functions of the distances between their centres. Confining ourselves to the case where there is no obvious separation of electron charges, as there is none in th crystals described above, it is clear that we must think of the molecules as bodi of very definite form. These attachments to one another are made at defini points and the forces there exerted may have very short ranges. The molecul are locked into crystalline structure, when attachments are made at sufficient point and the whole has the stability of an engineering structure. It may well be that i a liquid there are always some completed attachments, but insufficient in numb to give rigidity to the whole. In a gas there are no attachments at all.

I have to thank Mr. Shearer for his assistance in examining crystals by th powder method, which method he has greatly improved ; and Mr. J. Reid, f the labour and skill which he has devoted to making models.

Part of the apparatus employed was obtained through a grant from th Department of Scientific and Industrial Research.

I should also like to thank the General Electric Company of America for the great kindness in presenting me with Coolidge X-ray bulbs of special design.

Dr. C. CHREE, in moving a vote of thanks, said that the President's researches had open up fascinating vistas of investigation, which should lead to momentous discoveries.

Sir R. ROBERTSON, expressing the hope that these accurate measurements and brillia speculations would in time be applied to the compounds of nitrogen, seconded the vote of thank which was carried by acclamation.

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